

MATHEMATICS

(CLASS VII)

ARITHMETIC, ALGEBRA & GEOMETRY

BY

West Bengal Board of Secondary Education



Published By

WEST BENGAL BOARD OF SECONDARY EDUCATION



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(CLASS VII)

ARITHMETIC, ALGEBRA & GEOMETRY

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Only approved textbook in Mathematics for Class VII
for use in all recognised secondary schools under
West Bengal Board of Secondary Education



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West Bengal Board of Secondary Education

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FOREWORD

Revised syllabus of Mathematics was introduced in Class VI in the academic year 1981 and it was introduced in Class VII in 1982. In the years to come new syllabuses will be gradually introduced in Class VIII, IX and X successively. Teachers of secondary schools, degree colleges, teachers' training colleges, universities and some renowned specialists in mathematics shared their thoughts in preparing these syllabi. During the preparation of syllabus it was gradually felt that the publishing of Board's own textbooks was a necessity. The Board realised the importance of the opinion and decided to get mathematics textbooks written on its own initiative. This decision of the Board had its approval from the Government of West Bengal. Consequently, the responsibility of writing textbooks was given by the Board to mathematicians connected with mathematics education at different levels.

The Govt. of West Bengal distributed the Board's textbooks on mathematics, free of cost, among the students of Class VI and thus launched an admirable endeavour in preparing books for free distribution among students of the secondary stage. This effort of the State Government deserves congratulations. The Board has found, with pleasure, that the Bengali version of the books for classes VI and VII has been well received by teachers, educationist and mathematicians.

This year English version of the book for class VII is being published. In this book also the writers have sought to give lucid explanations of different topics of the syllabus. They have taken care that the methods of working out sums become free from being mechanical and made more interesting. Through different problems they have also tried to relate the topics with situations in life. This, in turn, will increase the eagerness for learning mathematics.

Mental abilities of the students have been reckoned while writing the book. The Board feels that objectives of the syllabus have been properly reflected in the book.

The Board requests mathematics teachers, educationists and all others to bring to its notice any demerit or flaw in the book and to offer constructive suggestions.

Like its Bengali version, this book for class VII has been published, on behalf of the Board, by Macmillan India Limited. In consideration of the financial condition of general students the book has been priced as low as possible. Finally, I, on behalf of the Board, gratefully acknowledge the work of the teachers' organisations and specialists in mathematics who helped in framing the syllabus and in writing the textbooks. I hope that books on mathematics published by the Board will make the subject more attractive to the students and will improve the standard of education in this discipline.

Shri Bhabesh Moitra
President

2nd March, 1983
Calcutta

West Bengal Board of Secondary Education.

CLASS VII—1982

Arithmetic

35 Marks

1. Revision of previous work only through exercises on topics introduced in earlier Classes.
2. Conversion of decimals (terminating and recurring) into fractions. Approximate value of fractions correct upto three places of decimals.
3. H.C.F. and L.C.M. of fractions and decimals. Applications to simple problems.
4. Square root by division. Applications to simple problems.
5. Application of Unitary method to simple problems relating to Time and Distance, and Simple Interest.
(Problems should be direct and properly graded).
6. Problems on square measures.

Algebra

35 Marks

1. The use of symbols to generalise simple arithmetical problems, without formally introducing equations.
2. Integral numbers : positive and negative. Basic operation : addition, subtraction, multiplication, and division.
3. Laws : Commutative, Distributive, and Associative.
4. Polynomials (with integral and fractional coefficients), their additions, and subtractions. Multiplication of two Polynomials, each with two terms. Division of Polynomials with a single term divisor.
5. The following formulae and their easy applications :
 $(a+b)^2 = a^2 + 2ab + b^2$, $(a-b)^2 = a^2 - 2ab + b^2$, $(a+b)(a-b) = a^2 - b^2$.
 Geometrical representation of the above formulae.
6. Simple factorisation using the above formulae, and laws stated in Sec. 3.
7. Formation of linear equations with one variable to represent simple problems, and their solutions. (Cases where the variable appears in the denominator are excluded).

Geometry

30 Marks

- The aim of teaching Geometry at this stage is the same as in Class VI.
1. To demonstrate that translation and rotation of solid bodies and also of geometrical plane figures are possible without any change of their size and shape. In case of geometrical plane figures, reflection and or paper folding may be used.
 2. Activities verifying the following statements :
 (i) When a straight line cuts two other straight lines, those other two straight lines are parallel if a pair of corresponding angles are equal.
 (ii) Two intersecting straight lines cannot both be parallel to a third straight line. (iii) Congruence of two triangles : SAS, AAS. Use of the symbol ($\cong$) to indicate congruence. (iv) In congruent circles (or in the same circle) equal chords cut off equal arcs and subtend equal angles at the centre, and conversely.
 3. Constructions : (i) Angle equal to a given angle, (ii) Triangles with given parts : (a) three sides, (b) two sides and the included angle, (c) one side and the two angles adjacent to it, (iii) Quadrilaterals with given parts.

CONTENTS

Arithmetic

	Page
Chapter 1	
Exercises on Previous Lesson ..	1-7
Chapter 2	
Decimal Fraction ..	8-24
Chapter 3	
H.C.F. and L.C.M. of Fractions ..	25-45
Chapter 4	
Square root ..	46-55
Chapter 5	
Unitary Method ..	56-65
Chapter 6	
Simple Interest ..	66-83
Chapter 7	
Area ..	84-99
Answers ..	100-104

Algebra

Chapter 1	
Use of Symbol ..	1-12
Chapter 2	
Whole Number ...	13-45
Chapter 3	
Laws ...	46-56
Chapter 4	
Polynomials ..	57-74
Chapter 5	
Algebraic Formulae and their easy applications ..	75-95
Chapter 6	
Simple Factorisation ..	96-101

Chapter 7

Forming Simple Equation and its solution	..	102-111
Answers	..	112-118

Geometry**Chapter 1**

Translation, Rotation and Reflection	..	1-20
--------------------------------------	----	------

Chapter 2

Verification of Some Geometrical Properties	..	21-37
---	----	-------

Chapter 3

Various Constructions	..	38-59
Answers	..	60

ARITHMETIC



Chapter 1

Exercises on Previous Lesson

Exercise 1

1. Find whether the following numbers are primes :

899, 1103.

2. Find the L. C. M. and H. C. F. of the following numbers :

396, 704, 710.

3. (i) What are the factors of 105?

(ii) Write down a pair of prime factors of 105.

(iii) What are the common factors of 105 and 165?

Which of these common factors is the H. C. F. of 105 and 165 and why?

4. (i) Write down the ten consecutive multiples of 42 starting from the smallest.

(ii) Write down 10 smallest multiples of 105.

(iii) Which of the above multiples of 42 and 105 are common? Which of these common multiples is the L. C. M. of 42 and 105 and why?

5. (i) What are the greatest and smallest numbers of five digits, one of whose common divisors is 73?

(ii) Is 73 the H. C. F. of the above two numbers?

6. What is the largest number of students among whom 75 blackberries and 60 lichies can be equally distributed?

7. What is the greatest number which when divides 14490 and 31530 leaves a remainder 6 in each case?

8. Students present on the Sports day could be arranged in 4, 5, 6, 9 or 10 equal rows. What is the lowest number of students? What will be the number of students if it lies between 300 and 400?

9. The circumferences of the larger and smaller wheels of a car are $3\frac{1}{2}$ m. and $2\frac{1}{2}$ m. respectively. What is the smallest distance travelled by the car so that the number of revolutions of the

two wheels are whole numbers? In the above case find out the number of revolutions of each wheel.

[Hints : $3\frac{1}{2}$ m. = 350 cm. and $2\frac{1}{2}$ m. = 250 cm.]

10. What is the greatest number which divides 62, 86 and 134 leaving the same remainder in each case? What are the other numbers which divide the above three numbers leaving the same remainder?

Exercise 2

1. Arrange in ascending order of values :

(i) $5+2$, $5-2$, 5×2 , $5 \div 2$, 5^2

(ii) $\frac{3}{4}$, $\frac{5}{7}$, $\frac{6}{11}$

(iii) $1\frac{1}{2}$, $\frac{8}{9}$, $1\frac{2}{3}$

2. Find simplest values :

(i) $\frac{10752}{16880}$ (ii) $\frac{3\frac{1}{2}}{2\frac{4}{5}}$ (iii) $\frac{2}{3} + \frac{11}{18} - \frac{7}{6}$

(iv) $\frac{1}{3} - \frac{1}{6} - \frac{1}{7} - \frac{1}{8}$ (v) $2\frac{2}{3} - 2\frac{4}{5} + 1\frac{5}{8} - \frac{1}{18}$

(vi) $4\frac{1}{3} \times 1\frac{1}{8} \times 8\frac{2}{3}$ (vii) $\frac{5}{6} \div \frac{7}{8} \times \frac{1}{16} \div \frac{8}{9} \times 4\frac{2}{3}$

(viii) $1\frac{2}{3}$ of $3\frac{3}{4} \div 5\frac{1}{2}$ of $5\frac{1}{2}$

(ix) $\frac{\frac{3}{4}(\frac{2}{3} \text{ of } 2\frac{5}{7} + \frac{1}{8} \text{ of } 1\frac{2}{3})}{\frac{2}{18} \times 1\frac{5}{8} \times \frac{6}{7} - \frac{1}{11}}$

(x) $\frac{\frac{1}{8}}{\frac{2}{9} \text{ of } 3 - \frac{1}{7} \times 5\frac{2}{3}}$

(xi) $9 - [1 \div \{2 - (1 + 1 - \frac{6}{11})\}]$

(xii) $\frac{1}{2 + \frac{1}{2 + \frac{1}{5 - \frac{1}{8}}}}$

3. $\frac{5}{8}$ ths of a bag of rice are sold. If the weight of the remaining quantity of rice is $40\frac{1}{2}$ kg., what was the weight of the full content of the bag?

4. Kamalbabu spends $\frac{2}{3}$ of his income for the family, keeps $\frac{1}{2}$ of his income for pocket expense and saves the rest. If his annual saving is Rs. 3840, what is his monthly income?

5. Monthly incomes of Raicharan, Haricharan and Brindaban as well as their expenses on food, clothing and

miscellaneous items as fractions of the respective monthly incomes are given in the following table. Fill in the blanks in the table by calculating their savings as fractions of their incomes :

Name	Monthly income (in Rupees)	As fractions of monthly income			
		Expenses on food	Expenses on clothing	Other expenses	Savings
Raicharan	400	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{4}$	
Haricharan	350	$\frac{3}{7}$	$\frac{2}{35}$	$\frac{2}{5}$	
Brindaban	570	$\frac{3}{8}$	$\frac{2}{15}$	$\frac{1}{6}$	

Calculate also the amounts of their expenses on food, clothing and miscellaneous items and of their savings and fill in the following table :

Name	Expenses			Savings (in Rupees)
	On food (in Rupees)	On clothing (in Rupees)	Others (in Rupees)	
Raicharan				
Haricharan				
Brindaban				

6. In an examination Maya and Chhaya have obtained $\frac{3}{8}$ and $\frac{1}{4}$ of the total marks respectively. Maya has received 20 marks more than Chhaya. What is the total mark? What are the marks obtained by each?

7. $\frac{1}{4}$ of the total money of a village panchayet is spent on the repairing of embankment and $\frac{1}{8}$ on the digging of ponds. 50 and 40 people are engaged for the first and the second work respectively. Each person engaged in the first work receives Rs. 10 more than each person in the second work. What is the total amount of money of the village panchayet? What sums were allotted for repairing of embankment and digging of ponds?

8. Some mangoes are to be distributed among Sulekha, Ayesha and Shyamali in such a way that Sulekha and Ayesha get $\frac{2}{5}$ and $\frac{3}{5}$ of the total mangoes respectively and Shyamali gets the rest. Find the least number of mangoes required.

Exercise 3

1. Arrange in ascending order of magnitude :

(i) $\cdot 69$, $\cdot 7$, $\cdot 077$, $\cdot 079$.

(ii) $\frac{2}{3}$, $\cdot 75$, $\frac{38}{50}$, $\cdot 781$.

2. Find out the simplest values :

(i) $2345\cdot 11 + 1982 + 736\cdot 8 + 4368\cdot 29 + \cdot 99 + 28\cdot 1$

(ii) $3\cdot 4 - \cdot 56789$ (iii) $4\cdot 56 \times \cdot 00234$

(iv) $816 \div \cdot 00008$ (v) $1\cdot 625 \div 62\cdot 5$

(vi) $\frac{8\cdot 7 \times 8\cdot 7 \times 8\cdot 7 - 1\cdot 3 \times 1\cdot 3 \times 1\cdot 3}{8\cdot 7 \times 8\cdot 7 + 8\cdot 7 \times 1\cdot 3 + 1\cdot 3 \times 1\cdot 3}$

3. Convert into decimal fractions :

$$\frac{31}{125}, \frac{11}{100}, \frac{3}{4}.$$

4. Having covered $\frac{1}{8}$ th of the total journey by train, $\frac{2}{3}$ rd by air, and $\frac{1}{12}$ th by ship, Dilip Babu found that he had 240 km. still to go. What was the length of his journey in kms? How much did he cover by air?

5. From the ration shop Surenbabu bought wheat, rice, sugar and flour. The quantity of rice was half that of wheat, of sugar half that of rice and flour half that of sugar. If the total quantity was 7.5 kg., what was the quantity of each?

6. 4 years ago the age of Mustak was $\frac{9}{7}$ of that of Hanif. Now the sum of their ages is 60 years. Find the present age of each.

7. Distribute Rs. 1220 among Alope, Abir and Ashis in such a manner that Abir gets $\cdot 5$ of Alope's share and Ashis gets $\cdot 05$ of Abir's share.

8. An exercise book contains 128 leaves of equal thickness. If the book is $\cdot 56$ cm thick, what is the thickness of one leaf?

9. A wheel rotates 1230 times to cover a distance of 2509.2 metres. What is the circumference of the wheel?

10. If $1 \text{ km} = \frac{1}{62} \text{ mile}$, convert 403 miles into kilometres.

11. The capacities of a bucket and a pot are 20 litres and 1.24 litres respectively. How many potfuls are to be poured into the bucket to fill it up? How much water will be left in the pot after the bucket has been filled up?

12. Some amount of money is distributed among Hasina, Amina, Sulekha and Sudeepta in such a manner that Hasina got $\frac{1}{25}$ of the whole money, Amina got $\frac{1}{4}$ of the rest, Sulekha got $\frac{1}{3}$ of what was left after paying those two and Sudeepta got at last the rest which was Rs. 2.40. What were the shares of Hasina, Amina and Sulekha?

13. $\frac{1}{4}$ and $\frac{2}{5}$ of a bamboo are in mud and water respectively and the rest 2.75 m is above water. What is the length of the bamboo?

14. A person donated $\frac{1}{2}$ of his savings to the local school and $\frac{1}{7}$ th of what remained to the library. If his donation to the school exceeds that to the library by Rs. 8400, what was his total saving?

Exercise 4

1. Find the square root :

(i) 576 (ii) 1024 (iii) 1521

2. A relief organisation resolved to raise contributions to help physically handicapped persons. It was decided that each member will pay as many rupees as the total number of members. The total fund thus raised was Rs. 441. What is the number of members of the organisation?

3. The students of a school raised Rs. 1156 as contributions for a festival. Each student contributed as many paise as the total number of students. What was the total number of students? What was the contribution of each student in rupees?

4. Find the smallest number by which 486 is to be multiplied to make the product a perfect square.

5. What is the smallest number by which 2187 is to be divided so that the quotient may be a perfect square? What is the square root of that perfect square?

6. Which smallest perfect square number is divisible by 2, 3, 4, 5 and 6?

7. The product of two numbers is 1156 and their quotient is 4. What are the two numbers?

8. A few people raised Rs. 65 by contributing twice as many twenty five paise and thrice as many five paise as the number of people. What was the number of people?

9. In arranging 1804 soldiers in a solid square, it was found that there were 40 soldiers too many. What was the number of soldiers in each row?

10. The greater of two numbers is $1\frac{1}{2}$ times the smaller one. If the product of these two numbers is 384, what are the numbers?

Exercise 5

1. 100 labours can dig a pond in 20 days. After 8 days' work 20 labours go away. In how many days will the rest of the labours finish the rest of the work.

2. 8 men can do a piece of work in 12 days. In how many days will the work be finished if 4 more men are engaged?

3. Half the work of a bridge construction was done in 24 days. After that 16 more men were engaged and the rest of the work was finished in 16 days. How many men were engaged at the beginning?

4. In the first instalment Shyamal made a profit of Rs. 54 by selling books worth Rs. 1200. In the second instalment he made a profit of Rs. 99 by selling books at the same rate as before. What was the total price of the books sold by him in the second instalment?

5. A person bought 80 litres of kerosene oil for Rs. 128. At the time of selling there was a wastage of 25 ml per litre of oil. He made a total profit of Rs. 48.28 by selling the rest of the oil. What was the selling price of oil per litre?

6. The profit by selling a watch for Rs. 199 is double the loss when it is sold for Rs. 175. What is the cost price of the watch?

7. At the beginning of the year the population of a certain village was 400. At the end of the year the population increased by 20. With the same rate of increment, what will be the population at the end of the second year?

8. Increment of the pay of a teacher is same for every year. After 6 years his pay will be Rs. 670 and after 12 years it will be Rs. 790. What will be his pay after 25 years?

9. After selling a radio for Rs. 935 there was a profit of 10 paise per rupee. What will be the profit or loss if the radio is sold for Rs. 810.

10. Kamalbabu travels 5 km in one hour. After every kilometre he takes rest for 5 minutes. What time would he take to travel 25 km?

Chapter 2

Decimal Fraction

2.1. Conversion of decimal fraction into ordinary fraction:

We know $\cdot 3 = \text{three tenths} = \frac{3}{10}$
 $\cdot 07 = \text{seven hundredths} = \frac{7}{100}$
 $\cdot 005 = \text{five thousandths} = \frac{5}{1000}$

Adding we get $\cdot 375 = \frac{3}{10} + \frac{7}{100} + \frac{5}{1000}$
 $= \frac{300}{1000} + \frac{70}{1000} + \frac{5}{1000} = \frac{375}{1000}$

Again, $2 \cdot 2 = 2 + \text{two tenths} = 2 + \frac{2}{10}$
 $\cdot 04 = \text{four hundredths} = \frac{4}{100}$
 $\cdot 006 = \text{six thousandths} = \frac{6}{1000}$

Adding we get $2 \cdot 246 = 2 + \frac{2}{10} + \frac{4}{100} + \frac{6}{1000}$
 $= 2 + \frac{200}{1000} + \frac{40}{1000} + \frac{6}{1000}$
 $= 2 \frac{246}{1000} = \frac{2246}{1000}$

Thus we find that converting $\cdot 375$ into ordinary fraction we get $\frac{375}{1000}$ and converting $2 \cdot 246$ we get $\frac{2246}{1000}$. From these two examples we get hints about a general rule. The rule is of the following type:

To convert any decimal fraction into ordinary fraction the number without the decimal point has to be written as the numerator and the denominator will be the number obtained by putting 'zero's (0)

after 1, the number of 0 being equal to the number of digits after the decimal point.

In order to express the ordinary fraction in the simplest form, it is necessary to arrive at the least values of the numerators and the denominators.

$$\text{Hence, } .375 = \frac{375}{1000} = \frac{3}{8}$$

$$\text{and } 2.246 = \frac{2246}{1000} = \frac{1123}{500} = 2\frac{123}{500}$$

2.246 can also be expressed as a compound fraction by keeping out the whole number 2 in the following way:

$$2.246 = 2 + .246 = 2 + \frac{246}{1000} = 2 + \frac{123}{500} = 2\frac{123}{500}$$

In converting decimal fractions, which are more than 1, into compound fractions, it is better to follow the second procedure as shown above.

Example 1. Find the simplest value by expressing in ordinary fraction and then express in decimal fraction:

$$(i) \frac{.0171}{.38}$$

$$\frac{.0171}{.38} = \frac{1.71}{38} = \frac{\frac{171}{100}}{38} = \frac{171}{100} \times \frac{1}{38} = \frac{9}{200} = .045$$

Briefly it can also be simplified as follows:

$$\frac{.0171}{.38} = \frac{171}{3800} = \frac{9}{200} = .045$$

[In this case the decimal points in the numerator and denominator have been shifted to the right by the same number of places in such a way that both the numerator and denominator have become whole numbers.]

(ii) Express 7 m 75 cm in metres.

$$7 \text{ m } 75 \text{ cm} = (7 + \frac{75}{100})\text{m} = (7 + \frac{3}{4})\text{m} = 7\frac{3}{4} \text{ m} \\ = 7.75 \text{ m}.$$

(iii) Express $\frac{4 \text{ litre } 185 \text{ ml}}{2 \text{ litre } 250 \text{ ml}}$ in the simplest decimal fraction.

$$\frac{4 \text{ litre } 185 \text{ ml}}{2 \text{ litre } 250 \text{ ml}} = \frac{4185}{2250} = \frac{837}{450} = \frac{93}{50} = \frac{186}{100} \\ = 1.86$$

Exercise 6

1. Express in simplest fraction :

(i) .26, .8, .07, 1.2, 2.05

(ii) .00275, .625, .256, .0085, 260.0025

(iii) $\frac{6.33}{.025}$, $\frac{.729}{.081}$, $\frac{.0099}{.0495}$, $\frac{12.144}{.0012}$, $\frac{7.007}{.00028}$

2. Express in lowest decimal fraction :

(i) $\frac{.02592}{.081}$ (ii) $\frac{1.5 \times 6.3}{7.2 \times 7.5}$ (iii) $\frac{\text{Re } 1 \text{ and } 69 \text{ P}}{\text{Re } 1 \text{ and } 4 \text{ P}}$

(iv) $\frac{5 \text{ litre } 265 \text{ ml}}{2 \text{ litre } 250 \text{ ml}}$ (v) $\frac{5 \text{ km } 11 \text{ m } 25 \text{ cm}}{3 \text{ km } 2 \text{ hm } 72 \text{ deci m}}$

(vi) $\frac{6 \text{ quintal } 55 \text{ kg } 50 \text{ g}}{1 \text{ quintal } 50 \text{ kg}}.$

3. (i) How many rupees are .225 of Rs. 15000?

(ii) How much is .01 of 15 quintal 25 kg?

(iii) How many rupees are .001 of .01 of Rs 1,00,000?

(iv) How much is .375 of 1500 km?

(v) If the price of .21 part of a property is Rs. 8400, what is the price of the whole property?

(vi) If .65 of 75 crores of people are illiterates, what is the total number of illiterates?

2.2. Conversion of recurring decimal fraction into simple fraction:

You have already learnt how to convert simple fraction into decimal fraction. In the previous section you have also learnt how to convert simple decimal fraction into simple fraction. Now, let us discuss how to convert recurring decimal fraction (e.g., $\cdot\dot{3}$, $\cdot 0\dot{1}$, $\cdot 1\dot{2}\dot{3}$, $\cdot 0\dot{2}1\dot{1}$ etc.) into simple fraction.

We know that

$$\cdot\dot{3} = \cdot 3333\ldots$$

$$10 \text{ times } \cdot\dot{3} = 3\cdot 333\ldots$$

Subtracting both sides of the first equality from the corresponding sides of the second equality, we get,

$$9 \text{ times } \cdot\dot{3} = 3\cdot 333\ldots \quad - \cdot 333\ldots$$

$$= 3$$

$$\therefore \cdot\dot{3} = \frac{3}{9}$$

Notice here that in place of $\cdot\dot{3}$ there is 3 in the numerator, there is just one 9 in the denominator and there is one digit in the recurring portion.

Here the main step in converting $\cdot\dot{3}$ into simple fraction is multiplication by 10. Let us now consider one more recurring decimal fraction, $\cdot\dot{1}\dot{2}$. Here multiplication by 10 will not serve the purpose.

$$\cdot\dot{1}\dot{2} = \cdot 121212\ldots$$

$$100 \times \cdot\dot{1}\dot{2} = 12\cdot 121212\ldots$$

Subtracting the first equality from the second equality, we get,

$$99 \times \cdot\dot{1}\dot{2} = 12$$

$$\therefore \cdot\dot{1}\dot{2} = \frac{12}{99} = \frac{4}{33}$$

In this case instead of multiplying by 10 we had to multiply by 100 or 10^2 . Thus we will have to multiply by such power of 10 that will once bring whole of the recurring part to the left of the decimal point. But this method will not always be applicable. When whole of the decimal part of a given number is recurring then it will be sufficient to multiply by 10 or any required power of 10. Let us consider another example:

$$\cdot\dot{1}2\dot{3} = \cdot 123123123123 \dots$$

$$1000 \times \cdot\dot{1}2\dot{3} = 123 \cdot 123123123 \dots$$

Subtracting,

$$999 \times \cdot\dot{1}2\dot{3} = 123$$

$$\therefore \cdot\dot{1}2\dot{3} = \frac{123}{999} = \frac{41}{333}$$

But when there is non-recurring part in the decimal part, the method has to be changed slightly. We take the following example:

$$\cdot 1\dot{2}3\dot{4} = \cdot 1234234234 \dots$$

$$10000 \times \cdot 1\dot{2}3\dot{4} = 1234 \cdot 234234 \dots$$

$$10 \times \cdot 1\dot{2}3\dot{4} = 1 \cdot 234234234 \dots$$

Subtracting, $(10^4 - 10) \times \cdot 1\dot{2}3\dot{4} = 1234 - 1 = 1233$

$$9990 \times \cdot 1\dot{2}3\dot{4} = 1233$$

$$\cdot 1\dot{2}3\dot{4} = \frac{1233}{9990} = \frac{137}{1110}$$

Here we have multiplied by 10^4 so that whole of the recurring part comes to the left of the decimal point. Then we have multiplied by 10 so that the non-recurring decimal part comes to the left of the decimal point. When we subtract after multiplying in the above manner, the recurring part is excluded.

In the same manner try to convert $1.2\bar{3}45$ into simple fraction.

2.3. Method of expressing as a formula:

In the previous section you have learnt what method should be applied to convert recurring decimal number into ordinary fraction. Now we shall try to express this method as a formula.

In the number $52.63\bar{7}51$ we have recurring part, non-recurring decimal part and the whole number part. If we are to bring once whole of the recurring part to the left of the decimal point, we shall have to shift the decimal point 5 places to the right i.e., we shall have to multiply the number by 10^5 . Multiplying this way, we get

$$10^5 \times 52.63\bar{7}51 = 5263751.751751751..... \quad (1)$$

Again, if we are to bring only the non-recurring decimal part of the number to the left of the decimal point, we shall have to shift the decimal point 2 places to the right i.e., we shall have to multiply the number by 10^2 . Multiplying this way, we get

$$10^2 \times 52.63\bar{7}51 = 5263.751751..... \quad (2)$$

Subtracting (2) from (1), we get

$$(10^5 - 10^2) \times 52.63\bar{7}51 = 5263751 - 5263$$

Dividing both sides by $10^5 - 10^2$, we get

$$\begin{aligned} 52.63\bar{7}51 &= \frac{5263751 - 5263}{10^5 - 10^2} \\ &= \frac{5263751 - 5263}{99900} \quad \dots \quad \dots \quad (3) \end{aligned}$$

This is the required fraction. This can be simplified by the method known to you and this we will do later. But let us clarify the method here.

From (3) we find that the numerator of the required fraction is the difference of two numbers. First one of these two numbers is **the number obtained by removing the decimal and recurring points from the given number**. The second one is **the number obtained by removing the decimal point from the non-recurring part of the given number**.

What do we find by analysing the denominator of the above fraction? In the denominator, we find that there are **three** consecutive 9's and the number of digits in the recurring part of the given number is also **three**. We notice further that after the three 9's there are **two** zeros and the number of digits in the non-recurring decimal part of the given number is also **two**. Thus we can write the formula as follows:

$$\begin{array}{l} \text{A recurring} \\ \text{decimal number} = \frac{\text{The number without decimal} \\ \text{and recurring points} - \text{Non-} \\ \text{recurring part of the number} \\ \text{without decimal point}}{9\text{'s as many as the number of} \\ \text{digits in the recurring part} \\ \text{followed by zeros as many as} \\ \text{the number of digits in the} \\ \text{non-recurring decimal part}} \end{array}$$

Let us now apply this formula in some examples.

Example 1. Convert $31\cdot5\bar{6}34\bar{9}$ into ordinary fraction.

$$\begin{aligned} 31\cdot5\bar{6}34\bar{9} &= \frac{3156349 - 315}{99990} \\ &= \frac{3156034}{99990} = \frac{1578017}{49995} = 31\frac{28172}{49995} \end{aligned}$$

We can also apply this formula for the examples of the previous section.

Example 2. Convert $\cdot1\bar{2}\bar{3}$ into ordinary fraction.

$$\cdot1\bar{2}\bar{3} = \frac{123 - 1}{990} = \frac{122}{990} = \frac{61}{495}$$

The following example is a special type:

Example 3. Convert $\cdot\bar{9}$ into ordinary fraction.

$$\cdot\bar{9} = \frac{9 - 0}{9} = \frac{9}{9} = 1.$$

Is not the answer a bit surprising?

Exercise 7

1. (a) Multiply each of $\cdot\bar{9}$, $\cdot\bar{5}$ and $\cdot\bar{8}$ by 10.

(b) Subtract each of given number from the corresponding results obtained above.

(c) In each case, how many times of the original number is the result of the above subtraction?

(d) From (b) and (c) write down the value of each given number in ordinary fraction.

2. (a) Multiply each of $\cdot1\bar{2}$, $\cdot\bar{3}\bar{5}$ and $\cdot\bar{7}\bar{8}$ by 100.

(b) Subtract each given number from the corresponding results obtained above.

(c) In each case, how many times of the original number is the result of the above subtraction ?

(d) From (b) and (c) write down the value of each given number in ordinary fraction.

3. (a) Multiply each of $\cdot 0\dot{3}$, $\cdot 1\dot{6}$ and $1\cdot 2\dot{4}$ by 100.

(b) Multiply each of them separately by 10.

(c) Subtract the values obtained in (b) from those in (a).

(d) In each case, how many times of the original number is the result of subtraction ?

(e) From (c) and (d) write down the values of each given number in ordinary fraction.

4. Express the following decimals in ordinary fractions with the help of formula:

(a) $\cdot \dot{3}$, $\cdot 0\dot{1}$, $\cdot 2\dot{7}$, $\cdot \dot{9}$, $\cdot 30\dot{3}$

(b) $\cdot 0\dot{8}$, $\cdot 1\dot{6}$, $\cdot 00\dot{7}$, $\cdot 21\dot{2}\dot{4}$, $\cdot 32\dot{1}\dot{5}$

(c) $\cdot 38\dot{1}4\dot{8}$, $\cdot 008\dot{4}\dot{5}$, $\cdot 21\dot{6}2\dot{5}$, $\cdot 002\dot{3}\dot{4}$

(d) $1\cdot 300\dot{9}$, $31\cdot 2\dot{5}$, $2\cdot 36\dot{8}$, $1\cdot 01\dot{2}0\dot{2}$

(e) $7\cdot 0\dot{2}\dot{8}$, $1\cdot 0\dot{7}\dot{2}$, $2\cdot 1\dot{0}\dot{1}$, $29\cdot 2\dot{4}30\dot{6}$

(f) $\cdot 3\dot{4}$, $\cdot 21\dot{9}\dot{7}$, $4\cdot 2\dot{6}$, $4\cdot 023\dot{4}\dot{1}$

2.4. Determination of approximate value:

If you want to divide 62 paise among five of your friends equally, each will get

$$62 \text{ P} \div 5 = 12\cdot 4 \text{ P.}$$

In a real situation, one may get 12 P but not $\cdot 4$ P. Thus the share of each will be either 12 P or 13 P.

If the share of each is taken as 12 P, each will get $(12\cdot 4 - 12) \text{ P} = \cdot 4 \text{ P}$ less.

Again, if the share of each is taken as 13 P, each will get $(13 - 12\cdot 4) \text{ P} = \cdot 6 \text{ P}$ in excess. In this case, it is necessary to determine the approximate

value of 12.4 to find out which one of 12 P or 13 P will be reasonable.

We find that 12 P is nearer to 12.4 P than 13 P. Thus it will be reasonable to take 12 P as approximate value of 12.4 P.

Again, if the price of 8 things is 62 P, how much is to be paid for buying one?

For one thing we shall have to pay

$$62 \text{ P} \div 8 = 7.75 \text{ P.}$$

In a real situation it is not possible to get .75 P. So we shall have to determine the price of each thing approximately.

If we pay 7 P for each thing, we pay $(7.75 - 7) \text{ P} = .75 \text{ P}$ less. And if we pay 8 P, we pay $(8 - 7.75) \text{ P} = .25 \text{ P}$ more. Thus we find that among 7 P and 8 P, 8 P is nearer to 7.75 P. So approximately the value of 7.75 P is taken as 8 P. Can you now write down the approximate values in paise of 24.65 P, 7.2 P, 9.06 P and 23.9 P?

In any compound decimal number, if the first digit after the decimal point is 5 or more, then the approximate whole number value of the given number will be the part of the given number before the decimal point + 1; if the first digit after the decimal point is less than 5 then the approximate value will be the given number without the decimal part.

In approximate whole number, the value of 25.37 is 25, the value of 4.89 is 5, the value of 6.39 is 6 etc.

In approximate rupees, the value of Rs. 15.89 is Rs. 16, the value of Rs. 36.25 is Rs. 36 etc.

In approximate kilometres, the values of 180.7 km and 45.45 km are 181 km and 45 km respectively.

Following is a list of areas of some states of our country—

<u>Name of State</u>	<u>Area (in sq. km.)</u>
West Bengal	87,900
Tripura	10,477
Kerala	38,864
Uttar Pradesh	2,93,800
Madhya Pradesh	4,43,400
Gujrat	1,87,000

In order to compare the areas of these States if we express them in units of 1000 sq. km, then it will be easier to arrange them in ascending or descending order. Fill in the following columns and compare:

<u>Name of State</u>	<u>Area in nearest sq. km.</u>	<u>Area in nearest 1000 sq. km.</u>	<u>Area in nearest 1,00,000 sq. km.</u>
West Bengal	87,900	88	1
Tripura	10,477		
Kerala	38,864		
Uttar Pradesh	2,93,800	294	3
Madhya Pradesh	4,43,400		
Gujrat	1,87,000		

Bigger the unit for finding approximate value, more is the difference between the actual value and the approximate value. As for example, if the area of Uttar Pradesh is taken as 3 lakh sq. km. instead of 294 thousand sq. km. then we will be committing more mistake. But we save time and labour by writing 3 lakh sq. km.; it also becomes easier for us to compare the areas of the States.

We decide the unit for finding approximate value according to necessity. We take 1 lakh km as unit if we want to measure the distance of other planets and satellites from the earth. The distance between different railway stations is measured in kilometres. Again we use nearest centimetres in measuring a piece of cloth. We use nearest kilogram in measuring coal; but we use one thousandth part of a gramme in measuring gold. In measuring gold there will be much loss if the approximate weight of 1.2346 gm is taken as 1 gm.

Thus there is a necessity of finding out approximate value upto one or two or three etc. decimal places. Let the length of the school playing field be 54.62 m. In approximate whole number its value will be 55 m and correct upto one decimal place its approximate value will be 54.6 m. Similarly, if the weight of a piece of gold is 3.5463 gm, then its approximate value correct upto one decimal place is 3.5 gm, correct upto two decimal places its value is 3.55 gm, correct upto three decimal places it is 3.546 gm. Thus, in short we may say, if the digit immediately after the number of places upto which the approximate value is to be found out be 5 or more

then the value of the digit immediately before this is to be increased by 1, otherwise the digit at that place will remain same. Thus, the approximate value of 3.4637 in nearest whole number is 3, correct upto one decimal place it is 3.5, correct upto two decimal places it is 3.46 and correct upto three decimal places it is 3.464. These approximate values may also be expressed symbolically as follows:

$$\begin{aligned}3.4637 &\approx 3 \text{ (in approximate whole number)} \\&\approx 3.5 \text{ (correct upto one decimal place)} \\&\approx 3.46 \text{ (correct upto two decimal places)} \\&\approx 3.464 \text{ (correct upto three decimal places)} \\&[\approx \text{means approximately equal to}] \end{aligned}$$

Again,

$$\begin{aligned}27.37648 &\approx 27.4 \text{ (correct upto one decimal place)} \\&\approx 27.38 \text{ (correct upto two decimal places)} \\&\approx 27.376 \text{ (correct upto three decimal places)} \end{aligned}$$

In the following table, values of some numbers upto one, two, three decimal places and their corresponding values correct upto one, two, three decimal places are given. Fill in the blanks in the table.

Decimal fractions	Number upto one decimal place	Correct upto one decimal place	Number upto two decimal places	Correct upto two decimal places	Number upto three decimal places	Correct upto three decimal places
4.3726	4.3	4.4	4.37	4.37	4.372	4.373
2.09182	2.0	2.1	2.09	2.09	2.091	2.092
.6982	.6	.7	.69	.70	.698	.698
25.84396						
.0818	0	.1	.08	.08	.081	.082
4.3216						
33.36812						
.0093718	0	0	0	.01	.009	.009

Note: In case of rupees we are to find out the approximate values correct upto two decimal places in all the cases as there is no real existence of rupees upto three decimal places. Thus Rs. 2.346 means Rs. 2 and 34.6 paise. But we cannot have .6 paise in a real situation.

Hence Rs. 2.346 = Rs. 2 and 34.6 paise.
 $\approx$ Rs. 2 and 35 paise.

When money is expressed in simple or decimal fraction, it is desirable to express it in nearest paise.



Exercise 8

1. Write the approximate values of 4,59,03,961 taking as unit, one crore, one lakh, one thousand and one hundred.

2. Grants for higher education given to five States are given in the following table. Find out the corresponding values of there grants in nearest lakh, thousand and hundred rupees and fill in the blanks in the table.

Name of State	Grants in Rupees	Grants in nearest lakh rupees	Grants in nearest thousand rupees	Grants in nearest hundred rupees
Andhra Pradesh	2,79,09,708			
Assam	44,31,362			
Bihar	1,08,47,388			
Karnatak	1,19,39,530			
Tamil Nadu	2,46,34,452			

3. What is the whole number in 45.974639? What is approximate value of the number in whole numbers? What will be the values of the number upto one, two and three decimal places? What will be the approximate values of the number correct upto one, two and three decimal places?

4. Fill in the blanks of the following table:

Number	Value of the number						
	Whole number	Nearest whole number	Upto one decimal place	Correct upto one decimal place	Upto two decimal places	Correct upto two decimal places	Upto three decimal places
45.3068							
4.3647							
1.00074							
.67371							
8.54387							
.0615							
.00123							

5. Express the following fractions into decimals:

(i) $\frac{1}{8}$, (ii) $\frac{22}{7}$ (correct upto two decimal places), (iii) $\frac{1}{14}$ (correct upto one decimal place), (iv) $\frac{314}{3}$ (correct upto three decimal places).

6. If 1 mile = 1.6093 km, express 1 km in mile correct upto two decimal places.

7. The speed of light is 186000 miles per second and 1 mile = 1.6093 km. Express the speed of light in nearest km. per second.

8. If 1 inch = 2.54 cm., express 1 cm. in inch correct upto one decimal place.

9. If 180 degrees = $\frac{22}{7}$ radians, express 1 radian in degrees correct upto two decimal places.

10. If 1 pound sterling = Rs. 18.14, express Rs. 100 in pound sterling correct upto two decimal places.

11. (i) If Rs. 7.35 be equally distributed among 4 persons, what will be the share of each?

(ii) If Rs. 2 be distributed among 7 girls, what will be the share of each?

(iii) Rs. 50 is distributed equally among 3 boys and 4 girls. What is the total share of the boys and that of the girls? Is the sum of these two shares equal to Rs. 50? If not, is it less or more than Rs. 50 and by how much?

Chapter 3

H. C. F. and L. C. M. of Fractions

3.1. Factor and Multiple:

If a number is divisible by another number, the divisor is called factor and the dividend is called multiple. Here by the word 'divisible' we mean that the quotient is a whole number and the remainder is zero. You have learnt all these in Class six—in case of whole numbers. We can extend these ideas in case of fractions also. First we shall discuss about factors in a greater detail.

3.2. Factors of a fraction:

If a fraction is divisible by another fraction, i.e., if after division the quotient is a whole number, then, as before, the dividend is called multiple and the divisor is called factor. For example, one factor of $\frac{4}{3}$ is $\frac{2}{9}$, for

$$\frac{4}{3} \div \frac{2}{9} = \frac{4}{3} \times \frac{9}{2} = 6 \text{ (whole number)}$$

However we may have other factors of $\frac{4}{3}$. They may be

$$\frac{2}{15}, \frac{2}{21}, \dots, \frac{4}{9}, \frac{4}{15}, \frac{4}{21}, \dots$$

You may verify that if we divide $\frac{4}{3}$ by any one of the above fractions, the quotient will be a whole number.

Now one may question, how can we find out the factors of a fraction, say, $\frac{4}{3}$? We should keep in mind that a factor should be such a fraction that when we divide the given fraction by this fraction, the quotient will be a whole number. For this the denominator of the required fraction has to be some multiple of the denominator of the given fraction and for the same reason the numerator of the required fraction has to be some factor of the numerator of the given fraction. We can express this by the following formula:

$\text{Factor of a fraction} = \frac{\text{A factor of numerator}}{\text{A multiple of denominator}}$

Again, we know that the number of factors of a whole number is finite and definite but the number of its multiples is infinite.

From the above formula we find that the number of factors of fraction is also infinite. This characteristic difference between the factors of a whole number and those of a fraction is worth noticing.

We should also pay attention to another aspect. Before applying the above formula to find out the factors, it is necessary to express the given fraction in the lowest form. Otherwise we may face complications. As for example, from the initial definition, $\frac{1}{9}$ is a factor of $\frac{4}{18}$, for

$$\frac{4}{18} \div \frac{1}{9} = 2 \text{ (whole number)}$$

But 9, the denominator of the factor $\frac{1}{9}$, is not a multiple, rather a factor of 18, the denominator

of the given fraction. To avoid complications of this type, we would express the given fraction in its lowest form before proceeding to find out its factors. Similar complications will also arise when we will find out the multiples afterwards. Thus throughout this chapter we will assume that the fractions are in their lowest forms and if they are not so they are to be expressed in their lowest forms at the very beginning.

Example 1. Find 6 factors of $\frac{2}{15}$.

Factors of 2 are 1 and 2. (1 can be taken as the factor of any number.) Multiples of 15 are 15, 30, 45, 60 and so on. Hence each of the following 6 fractions is a factor of $\frac{2}{15}$:

$$\frac{1}{15}, \frac{2}{15}, \frac{1}{30}, \frac{1}{45}, \frac{2}{45}, \frac{1}{60}.$$

It is possible to write many more factors. You can verify that if $\frac{2}{15}$ is divided by any one of the factors written above the quotient will be a whole number.

Example 2. Find 6 factors of each of the fractions $\frac{3}{7}$ and $\frac{5}{21}$.

The numerator of the first fraction is 3 and it has two factors—1, 3. The denominator of this fraction is 7 and its multiples are innumerable like 7, 14, 21, 28 etc. Then 6 factors of this fraction can be written as $\frac{1}{7}, \frac{1}{14}, \frac{1}{21}, \frac{3}{7}, \frac{3}{14}, \frac{3}{21}$. Check that if $\frac{3}{7}$ is divided by any one of these factors, the quotient is a whole number.

Again, the numerator 5 of the fraction $\frac{5}{21}$ has two factors—1, 5. The denominator of the fraction

is 21 and its multiples are innumerable like 21, 42, 63, 84 etc. Then we can easily write 6 factors as $\frac{1}{21}$, $\frac{1}{42}$, $\frac{1}{63}$, $\frac{5}{21}$, $\frac{5}{42}$, $\frac{5}{63}$. Divide and verify that each of these fractions is a factor of $\frac{5}{21}$.

Note: From the factors that have been written it is seen that $\frac{1}{21}$ is a factor of both $\frac{3}{7}$ and $\frac{5}{21}$. In other words, $\frac{1}{21}$ is a **common factor** of $\frac{3}{7}$ and $\frac{5}{21}$. There may be more common factors like this.

Example 3. Find 4 common factors of $\frac{35}{12}$ and $\frac{21}{16}$.

Factors of the numerator of $\frac{35}{12}$ are 1, 5, 7, 35 and the multiples of the denominator are 12, 24, 36, 48, 60, 72, 84, 96,.....; factors of the numerator of $\frac{21}{16}$ are 1, 3, 7, 21 and the multiples of its denominator are 16, 32, 48, 64, 80, 96,..... Among the factors of the two numerators we find that 1 and 7 are common factors. Among the multiples of the two denominators we find that 48 and 96 are common multiples. Thus $\frac{1}{48}$, $\frac{7}{48}$, $\frac{1}{96}$, $\frac{7}{96}$ are 4 common factors of the two given fractions. Show that if $\frac{35}{12}$ and $\frac{21}{16}$ are divided by these 4 fractions the quotient in each case will be a whole number.

Note: Following this method it is possible to determine some common factors of not only 2 but also 3 fractions.

Example 4. Find 6 common factors of the three fractions $\frac{4}{5}$, $\frac{8}{9}$, $\frac{4}{9}$.

It is easy to see that common factors of the three numerators are 1, 2, 4. There is no more common factor. The L.C.M. of the three denomina-

tors is 45. Some other common multiples are 90, 135, 180 etc..... Hence the following 6 fractions are common factors of the given fractions:

$$\frac{1}{45}, \frac{2}{45}, \frac{4}{45}, \frac{1}{90}, \frac{1}{135}, \frac{2}{135}.$$

Verify that each of these fractions is a common factor of the given fractions.

Exercise 9

1. Find two factors of each of the following fractions :

(i) $\frac{3}{8}$

(ii) $\frac{5}{7}$

(iii) $\frac{4}{11}$

(iv) $2\frac{1}{8}$

(v) $3\frac{1}{4}$

(vi) $1\frac{3}{8}$

(vii) $1\frac{7}{12}$

(viii) $\frac{9}{16}$

2. Find two common factors of each of the following pairs of fractions:

(i) $\frac{3}{4}, \frac{2}{8}$

(ii) $\frac{5}{6}, \frac{3}{8}$

(iii) $1\frac{2}{3}, 3\frac{3}{4}$

3.3. Method of finding common factors:

It is difficult to understand the method of finding out common factors from the discussion and examples of the previous section. It is necessary to find out any one common factor of the numerators of the given fractions. Then it is necessary to find out any one common multiple of their denominators. The fraction obtained by taking the above common factor as its numerator and the above common multiple as its denominator is a common factor of the given fractions. This method can be written in the form of the following formula:

$= \frac{\text{A common factor of some fractions}}{\text{a common factor of the numerators}} \\ \text{a common multiple of the denominators}$

Example 1. Find a common factor of the three fractions $\frac{4}{5}$, $\frac{5}{8}$, $\frac{2}{3}$.

The numerators— $4=1.2.2$, $5=1.5$, $2=1.2$. Thus we find that the common factor of the three numerators, 4, 5, 2, is 1 and there is no other common factor.

The denominators— $5=1.5$, $6=1.2.3$, $3=1.3$; the L. C. M. of these is $1.2.3.5=30$. Other common multiples are 60, 90, 120 etc.

Hence one common factor of the given fractions is $\frac{1}{30}$. Other common factors are $\frac{1}{60}$, $\frac{1}{90}$, $\frac{1}{120}$ etc. Since common multiples of the denominators are innumerable, it is evident that common factors of the given fractions will also be innumerable. Can you say which one of these is largest or which one is the Highest Common Factor (H.C.F.)?

We can have another method of finding out common factor. In Class VI you have learnt the method of converting some fractions into fractions having lowest common denominator. After converting them into fractions having lowest common denominator, the common factors of their numerators are to be found out. The fraction obtained by taking any one of these as numerator and this common denominator as denominator will be a common factor of the given fractions.

Example 2. Find one common factor of the two fractions $\frac{2}{5}$ and $3\frac{1}{2}$.

L. C. M. of the denominators of the two fractions is 10 and their forms with lowest common

denominator is $\frac{4}{10}$, $\frac{35}{10}$. Now, the only common factor of 4 and 35 is 1. Hence one common factor of the two given fractions is $\frac{1}{10}$.

Discussion: How many common factors of the given fractions can be determined by this method? This number will be definite and equal to the number of common multiples of the numerators that are obtained after the fractions are expressed with lowest common denominator. We can get as many common factors as we want by slightly changing the method.

After expressing the fractions with lowest common denominator it is possible to express them with common denominator which is not lowest. For instance, the pair $\frac{12}{30}$, $\frac{105}{30}$ is another form, with common denominator, of the fractions of the above example. Now, 1, 3 are the two common factors of the above two new denominators. From this we find that $\frac{1}{30}$ is another common factor of the two given fractions. In this way we can find out more common factors. But this method is not quite suitable for the determination of common factor.

3.4. Method of finding H. C. F. of fractions:

You have already learnt the method of finding out common factor of a number of fractions. You have also noticed that there are many such common factors. The largest of these is the highest common factor or H. C. F. Now the question is how to determine it?

If we analyse the method of finding out common factor a little more, we shall get the method of finding out H. C. F.

The formula for finding common factor of a number of fractions is

$$\frac{\text{a common factor of the numerators}}{\text{a common multiple of the denominators}}$$

We know that the value of a fraction is increased if the value of its numerator is increased or if the value of its denominator is decreased. It will be even better if these two alternative conditions are satisfied simultaneously. Thus to obtain the highest value of the common factor determined by the above formula, we shall have to take the highest value of the numerator and the lowest value of the denominator. Then the formula for H. C. F. will be as follows:

$\text{H.C.F. of fractions} = \frac{\text{H. C. F. of the numerators}}{\text{L.C.M. of the denominators}}$
--

Example 1. Find the H. C. F. of the three fractions, $\frac{2}{5}$, $1\frac{2}{3}$, $9\frac{1}{5}$.

The three fractions are

$$\frac{2}{5}, 1\frac{2}{3} = \frac{5}{3}, 9\frac{1}{5} = \frac{46}{5}$$

L. C. M. of the denominators of these fractions
= 15

and H. C. F. of their numerators = 1.

Hence, the required H. C. F. = $\frac{1}{15}$

It has been mentioned earlier that the method of expressing the fractions with common denominators is not very suitable for finding common factors. But this method is quite suitable for the determination of H. C. F.

Example 2. Find the H. C. F. of the three fractions, $5\frac{1}{5}$, $1\frac{2}{3}$, $2\frac{5}{6}$.

The three fractions are

$$5\frac{1}{5} = \frac{26}{5}, \quad 1\frac{2}{3} = \frac{5}{3}, \quad 2\frac{5}{6} = \frac{17}{6}.$$

The L. C. M. of the denominators of the three fractions = 30.

The forms of the three fractions with lowest common denominator are

$$\frac{156}{30}, \quad \frac{50}{30}, \quad \frac{85}{30}$$

and the H. C. F. of the numerators of these new fractions is 1.

Hence, the H. C. F. of the given fractions = $\frac{1}{30}$.

Exercise 10

1. Find five factors of each of the following pairs of fractions:

- (i) $\frac{3}{8}$, $\frac{7}{10}$ (ii) $3\frac{1}{2}$, $2\frac{1}{8}$ (iii) $\frac{9}{16}$, $\frac{14}{20}$
 (iv) $\frac{14}{18}$, $1\frac{5}{9}$ (v) $2\frac{5}{8}$, $5\frac{3}{8}$ (vi) $1\frac{2}{3}$, $2\frac{1}{2}$.

Is there any common factor among the factors of any of the above pairs? If so, find them out.

2. In each of the following cases find six common factors:

- (i) $2\frac{5}{8}$, $9\frac{1}{2}$, $4\frac{1}{8}$ (ii) $\frac{7}{4}$, $\frac{35}{36}$, $\frac{14}{9}$
 (iii) $\frac{8}{18}$, $1\frac{2}{3}$, $\frac{3}{5}$ (iv) $3\frac{1}{4}$, $2\frac{5}{14}$, $1\frac{3}{4}$
 (v) $9\frac{1}{8}$, $6\frac{9}{10}$, $5\frac{1}{2}$

3. Find H.C.F.:

- (i) $\frac{5}{12}$, $\frac{8}{15}$ (ii) $1\frac{1}{4}$, $1\frac{7}{8}$ (iii) $\frac{2}{3}$, $\frac{3}{27}$ (iv) $\frac{8}{9}$, $\frac{10}{21}$
 (v) $\frac{3}{10}$, $\frac{4}{15}$, $\frac{8}{20}$ (vi) 8 , $2\frac{2}{5}$, $\frac{8}{10}$
 (vii) $3\frac{1}{4}$, $\frac{3}{5}$, $\frac{9}{10}$ (viii) $2\frac{1}{8}$, $1\frac{7}{8}$, $\frac{7}{10}$.

3.5. Multiple of fractions:

We found the factors of $\frac{4}{3}$ in section 3.2. Now we shall try to find out the multiples of that fraction. Multiple of $\frac{4}{3}$ is such a fraction which when divided by $\frac{4}{3}$ gives a whole number as quotient. Thus the numerator of that fraction will be a multiple of the numerator of the given fraction that is of 4 and its denominator will be a factor of the denominator of the given fraction that is of 3. Multiples of 4 are 4, 8, 12, 16 etc. Factors of 3 are 1 and 3. Thus, $\frac{4}{1}, \frac{8}{1}, \frac{12}{1}, \frac{16}{1}, \dots$ etc. i.e., 4, 8, 12, 16, ...etc. and $\frac{4}{3}, \frac{8}{3}, \frac{12}{3}, \dots$ etc. are all multiples of $\frac{4}{3}$. From this we find that any fraction obtained by multiplying $\frac{4}{3}$ by a whole number is a multiple of $\frac{4}{3}$. This is but natural and this is the easiest meaning of the word, multiple.

Since there is a possibility of misunderstanding, this meaning in the case of fraction, there is a necessity of the method of determining multiples as discussed above. Not only $\frac{4}{3}$, the multiple of any fraction can be found out by the following formula:

$\text{Multiple of a fraction} = \frac{\text{any multiple of numerator}}{\text{any factor of denominator}}$

... (1)

Example 1. Find some multiples of $\frac{7}{6}$ by multiplying by whole numbers and by applying the formula.

Multiplying by whole numbers—

$$\frac{7}{6} \times 1 = \frac{7}{6}, \quad \frac{7}{6} \times 2 = \frac{7}{3}, \quad \frac{7}{6} \times 3 = \frac{7}{2}, \quad \frac{7}{6} \times 4 = \frac{14}{3} \text{ etc.}$$

i.e., each of $\frac{7}{6}, \frac{7}{3}, \frac{7}{2}, \frac{14}{3}$ is a multiple of $\frac{7}{6}$.

Divide each of these by $\frac{7}{8}$ and see that in each case the quotient is a whole number.

Applying the formula, 7 is the numerator of the given fraction and its multiples are 7, 14, 21, 28, 35,..... etc. 6 is the denominator and its factors are 1, 2, 3, 6. Hence, some multiples of $\frac{7}{6}$ are 7, 14, 21, $\frac{7}{2}$, $\frac{21}{2}$, $\frac{35}{2}$, $\frac{7}{3}$, $\frac{14}{3}$, $\frac{35}{3}$, $\frac{7}{6}$, $\frac{35}{6}$ etc. We can write many more multiples.

Verify, by dividing at least some of the above numbers and fractions by $\frac{7}{8}$, that the quotients are whole numbers.

Can you now say whether a factor of a fraction, given in its lowest form, can be a whole number? Can a multiple of such a fraction be a whole number?

3.6. Common multiple of fractions:

The formula of the previous section may not be very useful for finding multiple of one fraction but its changed form is quite useful in finding common multiple of a number of fractions. The changed formula will be as follows:

$$\begin{array}{l} \text{Any one multiple of a number of fractions} \\ = \frac{\text{Any one common multiple of the numerators}}{\text{Any one common factor of the denominators}} \end{array}$$

... (2)

Example 2. Find some common multiples of the fractions $\frac{7}{8}$, $\frac{9}{2}$, $1\frac{1}{6}$.

The three fractions are $\frac{7}{8}$, $\frac{9}{2}$, $\frac{7}{6}$.

The L. C. M. of the three numerators of the fractions = 63 and some of its common multiples are 63, 126, 189 etc.

The denominators are 8, 2, 6 and their common factors are only two— 1, 2.

∴ Some common multiples of the three given fractions are

$$63, \frac{63}{2} = 31\frac{1}{2}, \frac{63}{3} = 21.$$

Divide anyone of the above fractions by any one of the given fractions and see that in each case the quotient is a whole number.

Exercise 11

1. Multiply by whole numbers and find four smallest multiples of each of the following fractions:

$$(i) \frac{3}{4} \quad (ii) \frac{5}{7} \quad (iii) \frac{8}{9} \quad (iv) \frac{9}{10} \quad (v) \frac{5}{18} \quad (vi) \frac{14}{3}.$$

2. With the help of formula find five multiples of each of the above fractions.

3. Find three smallest common multiples in each of the following cases:

$$(i) \frac{2}{3}, \frac{4}{5} \quad (ii) \frac{3}{8}, \frac{7}{10} \quad (iii) 2\frac{1}{4}, 3\frac{3}{8}$$

$$(iv) \frac{7}{8}, \frac{8}{9} \quad (v) \frac{2}{3}, \frac{9}{10}, \frac{13}{16} \quad (vi) \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$$

$$(vii) 1\frac{1}{2}, 2\frac{1}{3}, \frac{3}{4} \quad (viii) \frac{14}{15}, 5\frac{3}{8}, 6\frac{3}{10}.$$

3.7. L. C. M. of fractions:

You have learnt the method of finding common multiples of fractions. You have also seen that one can have a number of such common multiples. The smallest or least of these common multiples is the lowest common multiple or L. C. M.

From formula (2), we get

$$\text{Common multiple} = \frac{\text{one common multiple of the numerators}}{\text{one common factor of the denominators}}$$

There will be a number of values of this fraction. The smallest value of this fraction will be obtained when the value of numerator is smallest and the value of the denominator is greatest. This is the method of reducing the value of a fraction. Thus the formula for finding the L. C. M. of a number of fractions is

$$\text{L. C. M. of some fractions} = \frac{\text{L. C. M. of numerators}}{\text{H. C. F. of denominators}}$$

Example 1. Find the L. C. M. of the three fractions, $\frac{6}{25}$, $\frac{8}{15}$, $2\frac{2}{5}$.

The three given fractions are $\frac{6}{25}$, $\frac{8}{15}$, $\frac{12}{5}$.

The L. C. M. of the numerators i.e., of 6, 8, 12 = 24 and the H. C. F. of the denominators i.e. of 25, 15, 5 = 5.

Hence the required L. C. M. = $\frac{24}{5} = 4\frac{4}{5}$.

Exercise 12

1. Find the L. C. M. of the following:

- | | | |
|--|--|--|
| (i) $\frac{3}{4}$, $\frac{9}{18}$ | (ii) $\frac{3}{5}$, $\frac{9}{20}$ | (iii) $\frac{8}{15}$, $\frac{10}{28}$ |
| (iv) $\frac{7}{8}$, $\frac{9}{32}$ | (v) $\frac{8}{9}$, $7\frac{1}{2}$ | (vi) $2\frac{3}{4}$, $3\frac{1}{2}$ |
| (vii) $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$ | (viii) $9\frac{1}{8}$, $6\frac{9}{10}$, $5\frac{1}{2}$ | |
| (ix) $\frac{7}{8}$, $8\frac{2}{3}$, $1\frac{2}{3}$ | (x) 3 , $\frac{3}{8}$, $1\frac{5}{4}$ | |
| (xi) 20 , $3\frac{3}{4}$, $6\frac{2}{3}$ | (xii) $\frac{8}{12}$, $1\frac{1}{4}$, $1\frac{8}{11}$ | |
| (xiii) $\frac{3}{4}$, $\frac{5}{8}$, $3\frac{9}{12}$ | (xiv) $5\frac{1}{4}$, $7\frac{1}{2}$, $1\frac{7}{8}$ | |
| (xv) $3\frac{1}{7}$, $2\frac{5}{14}$, $1\frac{9}{35}$, $4\frac{2}{7}$ | (xvi) $8\frac{2}{5}$, $7\frac{7}{8}$, $3\frac{1}{2}$ | |

3.8. Finding H. C. F. and L. C. M. of decimal fractions:

Let us suppose that we have to find the H.C.F. and L. C. M. of 4.5, .36, .009.

$$4.5 = \frac{45}{10}, \quad .36 = \frac{36}{100}, \quad .009 = \frac{9}{1000}$$

Now we shall have to express these fractions with lowest common denominator.

$$4.5 = \frac{45}{10} = \frac{4500}{1000}$$

$$.36 = \frac{36}{100} = \frac{360}{1000}$$

$$.009 = \frac{9}{1000}$$

You have already learnt the method of finding the H. C. F. and L. C. M. of fractions with lowest common denominators.

Now, the H. C. F. of the above fractions will be

$$\frac{\text{H. C. F. of } 4500, 360 \text{ and } 9}{1000}$$

$$\therefore \text{required H.C.F.} = \frac{9}{1000}$$

The L. C. M. of the fractions

$$= \frac{\text{L. C. M. of } 4500, 360, 9}{1000} = \frac{9000}{1000} = 9.$$

We can also apply the following method.

Example 1. Find the H. C. F. and L. C. M. of .5, .025, .45.

$$.5 = .500, \quad .025, \quad .45 = .450$$

Now the H.C.F. of 500, 25 and 450 = 25.

$\therefore$ required H. C. F. = $\cdot 025$

Again, L. C. M. of 500, 25 and 450 is 4500.

$\therefore$ required L. C. M. = $4\cdot 5$

Exercise 13

1. Find L. C. M. and H. C. F. of:

- (i) $1\cdot 6, 6\cdot 4$ (ii) $\cdot 23, 4\cdot 6$ (iii) $2\cdot 2, 3\cdot 3$
 (iv) $8, \cdot 08, \cdot 008$ (v) $4\cdot 9, \cdot 63, \cdot 056$ (vi) $\cdot 002, \cdot 04, \cdot 8$
 (vii) $12\cdot 5, 3\cdot 75, \cdot 175$. (viii) $\cdot 3, \cdot 009, 8\cdot 1, \cdot 00001$

3.9. Relationship between H. C. F. and L. C. M.:

In Class VI you have already seen that, in case of whole numbers, the product of the H. C. F. and L. C. M. of two numbers is equal to the product of the two numbers. Is this true in case of fractions also? Let us see.

From the formulae,

H. C. F. of two fractions

$$= \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$$

and their L. C. M.

$$= \frac{\text{L. C. M. of numerators.}}{\text{H. C. F. of denominators}}$$

Hence, for two fractions,

H. C. F. $\times$ L. C. M.

$$\begin{aligned} &= \frac{\text{H. C. F. of numerators}}{\text{L.C.M. of denominators}} \times \frac{\text{L. C. M. of numerators}}{\text{H.C.F. of denominators}} \\ &= \frac{\text{H. C. F. of numerators} \times \text{L. C. M. of numerators}}{\text{L.C.M. of denominators} \times \text{H.C.F. of denominators}} \\ &= \frac{\text{Product of the two numerators}}{\text{Product of the two denominators}} \\ &= \text{Product of the two fractions.} \end{aligned}$$

Thus we find that, like whole numbers, the product of the H. C. F. and L. C. M. of two fractions is equal to the product of the two fractions.

Example 1. Find H. C. F. and L. C. M. of the two fractions, $3\frac{1}{2}$, $2\frac{1}{8}$ and show that their product is equal to the product of the two fractions.

The two fractions are $\frac{7}{2}$, $\frac{17}{8}$.

H. C. F. of the two numerators = 1 and their L. C. M. = 119.

H. C. F. of the two denominators = 2 and their L. C. M. = 8

Hence, H. C. F. of the two fractions = $\frac{1}{8}$ and their L. C. M. = $\frac{119}{2}$.

$$\therefore \text{H. C. F.} \times \text{L. C. M.} = \frac{1}{8} \times \frac{119}{2} = \frac{119}{16} = 7\frac{7}{16}.$$

$$\text{Product of the two fractions} = \frac{7}{2} \times \frac{17}{8} = \frac{119}{16} = 7\frac{7}{16}.$$

Thus, the product of the H. C. F. and L. C. M. is equal to the product of the two fractions.

Like whole numbers, in the case of three fractions also, this result may not always be true.

Try to find out if the above result is true in case of any three fractions.

Example 2. Find H. C. F. and L. C. M. of the three fractions $1\frac{2}{3}$, $2\frac{5}{8}$, $9\frac{1}{5}$. Compare the values of the products of these H. C. F. and L. C. M. and of the three given fractions.

The three fractions are $\frac{5}{3}$, $\frac{17}{8}$, $\frac{46}{5}$.

H. C. F. of numerators = 1,

L. C. M. of denominators = 30.

Hence, H. C. F. of the three fractions = $\frac{1}{30}$

L. C. M. of numerators = 3910,

H. C. F. of denominators = 1.

Hence the L. C. M. of the three fractions = 3910.

H. C. F. $\times$ L. C. M. = $\frac{1}{30} \times 3910 = \frac{391}{3} = 130\frac{1}{3}$

Product of the three fractions.

$$= \frac{5}{3} \times \frac{17}{6} \times \frac{46}{5} = \frac{391}{3} = 130\frac{1}{3}.$$

Thus, the product of the H. C. F. and L. C. M. is greater than the product of the three fractions and they are not equal.

3.10. In the previous section we have learnt that, like whole numbers, the product of H. C. F. and L. C. M. of two fractions is equal to the product of the two fractions. Using this relation we can determine the H. C. F. of two fractions with the help of their L. C. M. and their L. C. M. with the help of their H. C. F.

Example 3. Find the L. C. M. of $9\frac{1}{5}$ and $\frac{9}{35}$, with the help of their H.C.F.

The fractions are $\frac{46}{5}$, $\frac{9}{35}$.

Their product = $\frac{46 \times 9}{5 \times 35}$

H. C. F. of the two numerators = 1,

L. C. M. of the two denominators = 35.

H. C. F. of the two fractions = $\frac{1}{35}$

$$\therefore \text{their L. C. M.} = \frac{46 \times 9}{5 \times 35} \div \frac{1}{35} = \frac{46 \times 9}{5} = \frac{414}{5} = 82\frac{4}{5}$$

Similarly, H. C. F. can be determined with the help of L. C. M. But this method is not applicable

in case of three fractions, because the product of three fractions is not equal to the product of their H. C. F. and L. C. M. But there is some relationship between the H. C. F. and L. C. M. of three or more fractions. In the next section this relationship will be discussed in brief.

3.11. Reciprocals:

The fraction obtained by interchanging the numerator and denominator of a fraction, i.e., by writing denominator for numerator and numerator for denominator, is called the reciprocal of the given fraction. We can also say that the first fraction is the reciprocal of the second. It can also be said that the two fractions are reciprocals of each other. It is easily understood that the product of a fraction and its reciprocal is 1. This is main property of reciprocal.

As for example, reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ and that of $\frac{3}{2}$ is $\frac{2}{3}$; $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals of each other and $\frac{2}{3} \times \frac{3}{2} = 1$.

Now we will compare the L. C. M. and H. C. F. of some fractions with those of their reciprocals.

Let us take four fractions $\frac{4}{5}$, $\frac{2}{15}$, $\frac{3}{7}$, $\frac{5}{28}$.

H.C.F. of the numerators = 1 and their L. C. M. = 60

H. C. F. of the denominators = 1 and their L. C. M. = 420.

H. C. F. of the four fractions

$$= \frac{\text{H. C. F. of numerators}}{\text{L. C. M. of denominators}} = \frac{1}{420};$$

$$\text{Their L. C. M.} = \frac{\text{L. C. M. of numerators}}{\text{H.C.F. of denominators}} = \frac{60}{1} = 60.$$

The reciprocals of the four fractions are $\frac{5}{4}$, $\frac{15}{2}$, $\frac{7}{3}$, $\frac{28}{5}$.

H. C. F. of these numerators = 1 and their
L. C. M. = 420

H. C. F. of these denominators = 1 and their
L. C. M. = 60.

Hence, the H. C. F. of the reciprocals = $\frac{1}{60}$ and their L. C. M. = 420. We find that the H. C. F. of the reciprocals is the reciprocal of the L. C. M. of the original fractions and the L. C. M. of the reciprocals is the reciprocal of the H. C. F. of the original fractions.

The relationships are true not only in this particular case but also in general. We can easily prove that these relationships are true in general. In doing so we shall only have to remember that the numerator of the reciprocal is the denominator of the given fraction and vice versa, i.e., the denominator of the reciprocal is the numerator of the given fraction.

H. C. F. of the reciprocals

$$\begin{aligned}
 &= \frac{\text{H. C. F. of numerators of the reciprocals}}{\text{L. C. M. of denominators of the reciprocals}} \\
 &= \frac{\text{H. C. F. of denominators of the original fractions}}{\text{L. C. M. of numerators of the original fractions}} \\
 &= \text{Reciprocal of} \\
 &\quad \frac{\text{L. C. M. of numerators of the original fractions}}{\text{H. C. F. of denominator of the original fractions}} \\
 &= \text{Reciprocal of L. C. M. of the original fractions} \\
 &= \frac{1}{\text{L. C. M. of the original fractions}}
 \end{aligned}$$

Similarly, we can prove the converse i.e.,

L. C. M. of reciprocals

$$= \frac{1}{\text{H. C. F. of the original fractions}}$$

General formula:

H. C. F. of some fractions $\times$ L. C. M. of their reciprocals

$=$ L. C. M. of fractions $\times$ H. C. F. of their reciprocals

$= 1$.

Using this formula we can find out the L. C. M. of a number of fractions with the help of their H. C. F. and their H. C. F. with the help of their L. C. M.

Example 4. Find H. C. F. of the four fractions $\frac{2}{5}, 1\frac{2}{3}, 9\frac{1}{5}, \frac{7}{8}$.

The four fractions are $\frac{2}{5}, \frac{5}{3}, \frac{46}{5}, \frac{7}{8}$.

Their reciprocals are $\frac{5}{2}, \frac{3}{5}, \frac{5}{46}, \frac{8}{7}$.

L. C. M. of the numerators of the reciprocals is 120 and their H. C. F. is 1

Hence, L. C. M. of the reciprocals $= 120$

and the H. C. F. of the given fractions $= \frac{1}{120}$

Exercise 14

- Find L. C. M. and H. C. F. of each of the following pairs of fractions and show that in each case their product is equal to the product of the two given fractions:
 - $\frac{7}{8}, \frac{6}{14}$
 - $\frac{5}{6}, \frac{3}{18}$
 - $1\frac{3}{4}, \frac{2}{3}\frac{1}{2}$
 - $1\frac{3}{5}, \frac{15}{84}$
 - $\frac{4}{9}, \frac{15}{81}$
- In each case find H. C. F. and L. C. M. of the three fractions. Test if the product of these H. C. F. and L. C. M. is equal to the product of the three fractions:
 - $\frac{8}{9}, \frac{4}{15}, \frac{8}{25}$
 - $8, 2\frac{3}{8}, \frac{8}{21}$
 - $\frac{6}{11}, \frac{12}{33}, \frac{24}{55}$
 - $\frac{15}{16}, \frac{25}{36}, 1\frac{3}{2}$
 - $\frac{2}{3}, \frac{3}{8}, \frac{5}{7}$

3. Find L. C. M. using H. C. F.:
- (i) $\frac{1}{2}, \frac{2}{3}$, (ii) $\frac{8}{16}, \frac{1}{25}$, (iii) $\frac{3}{4}, \frac{9}{16}$
4. Find H. C. F. using L. C. M.:
- (i) $\frac{5}{12}, \frac{1}{2}, \frac{1}{3}$ (ii) $\frac{5}{6}, 7\frac{1}{2}$ (iii) $2\frac{3}{4}, 3\frac{5}{6}$
- (iv) $1\cdot28, \cdot432$ (v) $1\cdot92, \cdot729$.
5. L. C. M. and H. C. F. of two fractions are $6\frac{2}{3}$ and $\frac{2}{3}$ respectively. If one fraction is $3\frac{1}{3}$ what is the other?
6. Product of two fractions is $\frac{9}{8}\frac{7}{4}$, their L. C. M. is $2\frac{1}{4}$; what is their H. C. F.?
7. Product of two fractions is $\frac{1}{8}\frac{6}{1}$, their H. C. F. is $\frac{2}{7}$; what is their L. C. M.?
8. L. C. M. of two numbers is 3 and the numbers are $\cdot6$ and $\cdot75$; what is their H. C. F.?
9. H. C. F. of two numbers is $\cdot072$ and the numbers are $1\cdot8$ and $1\cdot152$; what is their L. C. M.?
10. (a) Find (i) L. C. M., (ii) H. C. F., (iii) reciprocals, (iv) L. C. M. of the reciprocals, (v) H. C. F. of the reciprocals of the two fractions $4\frac{1}{2}$ and $2\frac{1}{10}$.
 (b) (i) Now fill in the following table:

Numbers	L.C.M.	H.C.F.	Reciprocals	L. C. M. of Reciprocals	H. C. F. of Reciprocals

(ii) From the above table, fill in the following blanks:

- L. C. M. of the two numbers = — of the two reciprocals.
 H. C. F. of the two numbers = — of the two reciprocals.
 H. C. F. of the two reciprocals = — of the two numbers.
 L. C. M. of the two reciprocals = — of the two numbers.

Chapter 4

Square root

4.1. Finding square roots by the method of division:

Following are squares of some numbers:

$$1^2 = 1$$

$$9^2 = 81$$

$$10^2 = 100$$

$$99^2 = 9801$$

$$100^2 = 10000$$

$$999^2 = 998001$$

$$1000^2 = 1000000$$

From above we find that the square root of a number of one or two digits is a number of one digit, square root of a number of three or four digits is a number of two digits, square root of a number of five or six digits is a number of three digits, and so on.

Apart from this, you already know that

(i) if the digit in the unit's place of any number is 1 or 9, the digit in the unit's place of its square will be 1, for example, $11^2 = 121$, $9^2 = 81$;

(ii) if the digit in the unit's place of any number is 2 or 8, the digit in the unit's place of its square will be 4, for example $12^2 = 144$, $8^2 = 64$;

(iii) if the digit in the unit's place of any number is 3 or 7, the digit in the unit's place of its square will be 9 for example, $13^2 = 169$, $17^2 = 289$;

(iv) if the digit in the unit's place of any number is 4 or 6, the digit in the unit's place of its square will be 6, for example $14^2 = 196$, $16^2 = 256$;

(v) if the digit in the unit's place of any number is 5, the digit in the unit's place of its square will be 5, for example, $15^2 = 225$, $25^2 = 625$.

(vi) if the digit in the unit's place of any number is 0, the digit in the unit's place of its square will be 0.

There is some order in the above rules which can be seen from the following diagram:

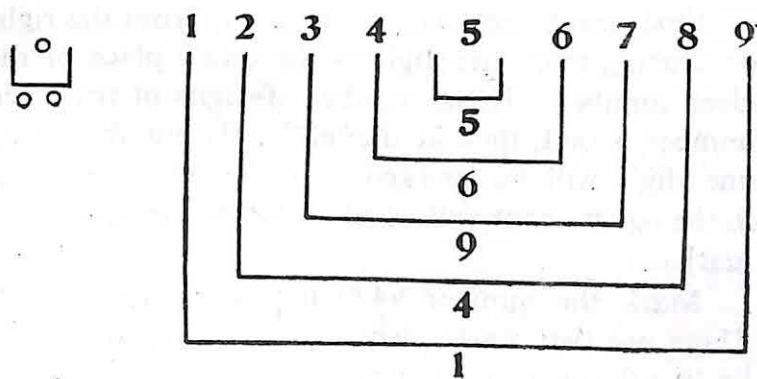


Fig. 1

The above six rules can be easily seen through this diagram. Now from this diagram or the above six rules, we easily get two more rules.

(vii) The digit in the unit's place of a perfect square will be 0, 1, 4, 5, 6 or 9.

(viii) The digit in the unit's place of any perfect square cannot be 2, 3, 7 or 8.

In Class VI you have learnt how to find out square root by factorisation. For example,
 $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

$\therefore$ Square root of $64 = 2 \times 2 \times 2 = 8$.

4.2. This method of finding factors is not very helpful when we want to find out square root of large numbers. Let us suppose that we shall have to find out the square root of 9409. In this case it is rather laborious to find out the factors of this number.

Square roots of large numbers can be found easily by the method of division.

First, mark each pair of the digits from the right or starting from the digit in the unit's place of the given number. If the number of digits of the given number is odd, then at the left of the number, only one digit will be marked. The number of digits in the square root will be the same as the number of marks.

Mark the number 9409 in the above method. There are two marks, which means that there will be two digits in the square root.

Notice that the first pair in the left is 94; 94 is a number between 9^2 and 10^2 . This means that the first digit from the left or the digit in the ten's place of the required square root is 9. Now put the mark for division to the right of 9409 and write 9. Write 81, the square

$$\begin{array}{r} 94\overline{09}(9 \\ 81 \\ \hline 13 \end{array}$$

$$\begin{array}{r} 94\overline{09}(9 \\ 81 \\ \hline 18)1309 \end{array}$$

of 9 below 94 and subtract. The result of subtraction is 13. Write the next pair 09 to the right of 13. Then you get 1309.

Put a mark for division to the left of 1309. Here, in the place of divider, write $18 (=9 \times 2)$. One largest digit is to be put to the right of the divider 18 and the quotient 9. This digit should be such that its product with the whole divider will not be greater than the dividend 1309. This digit can be found by the method of trial and error. If we put 7 to the right of the divider 18 and of the quotient 9 and then multiply the divider 187 by 7, the product is 1309.

Required square root is 97.

Verify that $97^2 = 9409$

$$\begin{array}{r} \overline{94\,09}(97 \\ 81 \\ \hline 187)1309 \\ 1309 \\ \hline \end{array}$$

If the number, whose square root is to be found, is still larger, the method of division is to be continued in the same manner.

Example 1. Find the square root of 356409.

$$\begin{array}{r} \overline{35\,64\,09}(59 \\ 25 \\ \hline 1064 \end{array}$$

Step 1: Below 35, the first pair from left, we put 25 ($=5^2$) and subtract. The result is 10. Putting 64, the next pair to the right of 10, we get 1064.

$$\begin{array}{r} \overline{35\,64\,09}(59 \\ 25 \\ \hline 109)1064 \\ 981 \\ \hline 8309 \end{array}$$

Step 2: Putting a mark of division to the left of 1064 we take 10, double of the quotient 5 obtained above. As above we shall have to put the highest

digit to the right of this 10 and of the quotient 5. By inspection we find that this digit is 9. If we put $109 \times 9 = 981$ below 1064 and subtract, we get 83 and putting the last pair 09 to the right of 83 we get 8309.

Step 3: Putting $59 \times 2 = 118$ in the place of divisor to the left of 8309, as before, we shall have to find out the greatest digit to be put to the right of 118 and of 59. By inspection that digit is found to be 7. Thus at this step the number at the divisor is 1187 and multiplying it by 7 of the quotient we get exactly 8309. If we put this number below the dividend and subtract, the result is 0. At this stage the work of finding square root is over and the required square root is 597.

$$\begin{array}{r} \overline{35\ 64\ 09}(597 \\ \underline{25} \\ 109)\overline{1064} \\ \underline{981} \\ 1187)\overline{8309} \\ \underline{8309} \end{array}$$

Example 2. What is the square root of 651249 ?

$$\begin{array}{r} \overline{65\ 12\ 49}(8 \\ \underline{64} \\ 16)\overline{112} \end{array}$$

Here whatever digit is put to the right of 16 the number will be greater than 112. In this case it is not possible to divide as before. Thus we shall have to put 0 to the right of 16. In the quotient also we shall have to put 0 to the right of 8 and to put the next pair 49 to the right of 112. Now in the place of the divisor we have 160 ($= 80 \times 2$). Now, as before, if we put, by inspection, 7 to the right of 80 and multiply 1607 by

$$\begin{array}{r} \overline{65\ 12\ 49}(807 \\ \underline{64} \\ 1607)\overline{11249} \\ \underline{11249} \end{array}$$

7, the product is 11249. Hence, the required square root is 807.

Note: In the last step, we have written 'by inspection' before the digit 7. This has some reason. At this step, if for the time being, we ignore the last digit of the divisor 160, we get 16 and ignoring the last two digits of the dividend, 11249 we get 112, dividing 112 by 16 we get 7. That is why the digit in the quotient at this step is taken as 7. If it so happened that after putting 7 to the right of the divisor and of the quotient and multiplying the new divisor 1607 by 7 the product was greater than 11249, then instead of 7 we had to take 6. This is the method of finding any particular digit of the quotient.

Example 3. Find the square root of 1522756.

Note here that only one digit in the left has been marked.

$$\begin{array}{r}
 \overline{1}5227\overline{5}6(1234 \\
 \underline{1} \\
 22)52 \\
 \underline{44} \\
 243)827 \\
 \underline{729} \\
 2464)9856 \\
 \underline{9856}
 \end{array}$$

Example 4. What smallest number should be subtracted from 8956 so that the result is a perfect square? What smallest number should be added to this number so that the result is a perfect square?

Ans: If we try to find out the square root of 8956 by the method of division, we shall find that the remainder will be 120. Hence if we subtract 120 from 8956 the result will be a perfect square.

$\therefore 8956 - 120 = 8836 = (94)^2$. This number is a perfect square.

Thus the required smallest number is 120.

The perfect square immediately next to 8956 ($=94^2$) is $95^2 = 9025$.

Now, $9025 - 8956 = 69$

$\therefore$ if we add at least 69 with 8956, the result will be 9025 ($=95^2$) which is a perfect square.

$\therefore$ in the second case required smallest number is 69.

Note: From the above examples we find that the digits of the square root are determined one by one from the left. This also happens in the usual method of ordinary division. The digits of the quotient are determined one by one from the left. This is the difference between the methods of multiplication or addition and of division or finding square root. In the usual method the digits in the sum or difference are determined one by one from the right.

Example 5. 5000 soldiers are arranged in a square taking maximum number in the front row. How many soldiers are there in the front row and how many of them will be left out?

[Let us see how a group of persons are arranged in a square:

Here we have arranged 25 soldiers in a square. (The positions of the soldiers are marked by 'x'.)

x	x	x	x	x
x	x	x	x	x
x	x	x	x	x
x	x	x	x	x
x	x	x	x	x

Again we have arranged 16 plants in a square, marking their positions by 'x'.]

x	x	x	x
x	x	x	x
x	x	x	x
x	x	x	x

Here we first try to find out the square root of 5000 by the method of division.

The unfinished square root is 70 and the remainder is 100.

5000	(70
49	
140)	100

∴ maximum number of soldiers that will be in the front row is 70 and 100 soldiers will be left out.

Note: At the time of arranging in a square there will be equal number of i.e., 70 soldiers in each row. We notice, further, that the number of rows and columns are all 70.

Example 6. Each student of a school contributed half as many paise as the total number of students for the flood relief fund. If the total contribution is Rs. 1568, what is the total number of students? What is the amount of contribution of each?

If instead of paying half as many, each student would have paid as many paise as the total number

of students, then the total contribution would have been doubled i.e., $\text{Rs. } 1568 \times 2 = \text{Rs. } 3136 = 313600$ paise.

Now, the square root of 313600 is 560.

$\therefore$ total number of students is 560.

Contribution of each student $= (560 \div 2)$ paise
 $= 280$ paise.
 $= \text{Rs. } 2.80$

[Multiply Rs. 2.80 by 560 and verify if the result is Rs. 1568].

Note: If in finding the square root by the method of this chapter there is any remainder in the last step, the given number is not a perfect square.

Exercise 15

1. (a) Find the number of digits in the square root of each of the following numbers without actually finding its square root.

(i) 361 (ii) 93600 (iii) 1239251209.

(iv) 625686734489.

(b) What may be the digits in the unit's place of the square roots of the following numbers?

(i) 5561 (ii) 289 (iii) 5329.

2. Find square root by the method of division:

(i) 729 (ii) 961 (iii) 529 (iv) 625 (v) 1521

(vi) 6561 (vii) 1296 (viii) 1681 (ix) 63504

(x) 680625 (xi) 1004004

(xii) 48024900 (xiii) 100060009

(xiv) 189475225 (xv) 360117609604

3. What smallest number should be added to 123401 to make it a perfect square? What smallest number should be subtracted from the same number to make it a perfect square?

4. What smallest number should be added to or what smallest number should be subtracted from 9545 to make it a perfect square in each case?

5. Soldiers were arranged in a square with maximum number in the front row. Total number of soldiers was 11050. After being arranged in a square 25 soldiers were in excess. What was the number of soldiers in the front row?

6. 4096 plants were put in a square field at equal distance. How many plants were there in each row?

7. Each member of a club contributed as many 2 paise as the total number of members and the total contribution was Rs. 141.12. What was the number of members of the club and how much did each member contribute?

8. Which greatest and smallest numbers of four digits are perfect squares?

9. Which smallest number of six digits is a perfect square?

10. One number is twice another number. Their product is 2048. Find out the two numbers.

Chapter 5

Unitary Method

5.1. Time and Distance:

In Class VI you have learnt unitary method and its application in questions regarding profit and loss and time and work. Here you will learn its application in questions on time and distance.

Initially the questions on time and distance arise from travelling. Everybody has to travel. You have also seen different types of travelling. Some people walk, some go by cycle, some travel by bus, tram, train, boat, ship or aeroplane. Apart from human beings, reptiles and some other animals also travel in different places—on the land, on walls or in the forest. Fish or other aquatic animals travel in water and birds travel in air—some very high and some at a low height.

There are two common properties in all the different types of travelling described above. In every case some distance is covered and for this some time is required. These two are the common properties of travelling—**distance travelled and time required for that**. From the relationship of these two properties we get a new concept which is called **speed**.

It is a bit difficult here to define speed. But one point can be understood easily. There may be two types of speed—**uniform speed** and **non-uniform speed**. If a body always travels same distance in

a given time, we say that the body is travelling with uniform speed. If a car travels 60 km in one hour or 1 km in one minute or $\frac{1}{60}$ km in one second, then we may suppose that the car is travelling with uniform speed and its speed is 60 km per hour. It is also clear that 60 km per hour = 1 km per minute = $\frac{1}{60}$ km per second.

But if we find that a car travels 60 km in first hour, 70 km in second hour and 55 km in third hour, then it is clear that the car is not moving with uniform speed. We cannot discuss any more in this respect at this stage.

During the discussion that follows in this chapter we will assume that moving objects have all got uniform speed. In other words the term speed will stand for 'uniform speed'. We can apply unitary method in solving arithmetical problems on uniform speed.

Example 1. A cyclist travels 36 km in $4\frac{1}{2}$ hours in uniform speed. What is his speed? What distance will be covered by him in 5 hrs. if his speed is same? What time will he take to travel 60 km?

In $4\frac{1}{2}$ ($=\frac{9}{2}$) hours the cyclist travels 36 km
 „ 1 „ „ „ „ $36 \div \frac{9}{2}$ „
 $= 36 \times \frac{2}{9}$ km = 8 km.

Hence his speed is 8 km per hour or 8 km/hr. In 1 hr. if he travels 8 km, then in 5 hrs. he will travel $8 \text{ km} \times 5 = 40 \text{ km}$.

Again, to travel 8 km, the time required is 1 hr.
 „ „ 1 km „ „ „ $\frac{1}{8}$ hr.
 „ „ 60 km „ „ „ $\frac{1}{8} \times 60$ hr.
 $= \frac{15}{2}$ hrs. = $7\frac{1}{2}$ hrs.

Again, same speed can be expressed in different units and this can also be done by unitary method. As for example, in the above example, the speed of the cyclist is 8 km/hr. which means that the distance travelled in 1 hr or 60 minutes is 8 km. Hence the distance travelled in 1 minute is $\frac{8}{60}$ km = $\frac{2}{15}$ km = $(\frac{2}{15} \times 1000)$ m = $\frac{400}{3}$ m = $133\frac{1}{3}$ m.

Hence, we can say that in metre/minute unit, the speed of the person is $133\frac{1}{3}$ metres per minute or, in short, $133\frac{1}{3}$ m/min. Thus, the two speeds 8 km/hr and $133\frac{1}{3}$ m/min. are same.

Again the distance covered in 1 min (=60 secs) is $\frac{400}{3}$ metres.

Hence, the distance covered in 1 sec is $\frac{400}{3 \times 60}$ m

$$\text{or, } \frac{400 \times 100}{3 \times 60} \text{ cm} = \frac{2000}{9} \text{ cm} = 222\frac{2}{9} \text{ cm.}$$

Thus, in centimetre-second unit, the speed is $222\frac{2}{9}$ centimetres per second or, in short, $222\frac{2}{9}$ cm/sec.

From the above discussion, we may say that the following three speeds are equal.

$$8 \text{ km/hr} = 133\frac{1}{3} \text{ m/min} = 222\frac{2}{9} \text{ cm/sec.}$$

Formula for speed:

From the above discussion and from the unitary method of finding speed we may write a formula for uniform speed in the following manner:

$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time}} \quad \dots (1)$
--

We easily get the two following corollaries from the above formula:

$$\boxed{\text{Distance travelled} = \text{Speed} \times \text{time}} \quad \dots (2)$$

and $\boxed{\text{Time} = \frac{\text{Distance travelled}}{\text{Speed}}} \quad \dots (3)$

From these three relations, given any two of time, distance and speed, we can find out the third.

Example 2. In the following table any two of time, distance and speed are given, find out the third. Write proper unit in each case.

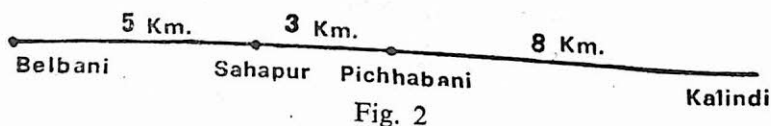
Time	Distance travelled	Speed
2 mins	50 metres	$(50 \div 2)\text{m/min}$ $= 25\text{m/min.}$
3 hrs	87 km	
4 secs		10m/sec.
$2\frac{1}{2}$ hrs		20 km/hr.
	50 km	10 km/hr.
	108 metres	9m/sec.
8 mins	256 metres	

Example 3. The speeds of a few cars are given

in the following table. Express them in required units and write in the appropriate place.

Speed per hr.	Speed per min.	Speed per sec.
75 km	1250 m	$2083\frac{1}{3}$ cm.
15 km		
50 km		

Example 4. The distances between Belbani and Sahapur, Sahapur and Pichhabani, and Pichhabani and Kalindi are 5 km, 3 km and 8 km respectively. A man starts from Belbani at 7 a.m. for Kalindi. If he travels 3km/hr when will he reach Sahapur, Pichhabani and Kalindi?



The speed of the man is 3 km per hour. The distance between Belbani and Sahapur is 5 km.

To travel 3 km he takes 1 hr or, 60 min.

" " 1 " " " $\frac{60}{3}$ min = 20 min.

" " 5 " " " (20×5) min = 100 min
= 1 hr 40 min.

He will reach Sahapur at 7 hr + 1 hr 40 min
= 8 hr 40 min or 8.40 a. m.

The distance between Sahapur and Pichhabani = 3 km. We already know that he takes 1 hr to travel 3 km.

Therefore, he will reach Pichhabani at 9.40 a.m.

Again, the distance between Pichhabani and Kalindi is 8 km.

To travel 8 km he will take (20×8) min = 160 min
= 2 hrs 40 min.

$\therefore$ He will reach Kalindi at 9hr 40 min + 2 hr 40 min. = 12 hr 20 min. or 12.20 p.m.

Example 5. The distances of Bandel, Burdwan, Durgapur, Asansol and Madhupur from Howrah are 40 km, 100 km, 160 km, 200 km and 280 km respectively. A train started from Howrah at 9 a.m. and reached Madhupur at 12.50 p.m. If the train stopped for 5 minutes at each of the four stations after Howrah, what was the speed of the train? When did the train reach Bandel and Burdwan? When did it leave the stations Durgapur and Asansol?

12.50 p.m.—9.00 a.m. = 3 hr 50 min.

At 4 stations the train stopped for $5 \text{ min} \times 4$
= 20 min.

3hr. 50 min—20 min. = 3hr. 30 min.

In 3hr 30min = $3\frac{1}{2}$ hrs the train travelled 280 km.

In 1 hr it went $(280 \div 3\frac{1}{2})$ km = $(280 \times \frac{2}{7})$ km
= 80 km.

$\therefore$ the speed of the train is 80 km/hr.

It goes 80 km in 1 hr.

“ “ 1 “ in $\frac{1}{80}$ hr.

“ “ 40 “ “ $\frac{40}{80}$ hr = $\frac{1}{2}$ hr = 30 min.

Since it left Howrah at 9.00 a.m., it reached Bandel at 9.30 a.m. and left Bendel at 9.35 a.m. To reach Burdwan it travelled $(100 - 40)$ km = 60 km

more. We have already seen that for 1 km it took $\frac{1}{80}$ hr, then for 60 km it took $\frac{60}{80}$ hr = $\frac{3}{4}$ hr = 45 min.

Hence, it reached Burdwan at 9 hr 35 min + 45 min = 10.20 a.m. and left that station at 10.20 a.m. + 5 min = 10.25 a.m. To reach Durgapur it had to travel $(160 - 100)$ km. = 60 km. more.

We have already seen that for 60 km it took 45 minutes.

Therefore, the train reached Durgapur at 10.25 a.m. + 45 min = 11.10 a.m. and left Durgapur at 11.10 a.m. + 5 min = 11.15 a.m.

To reach Asansol it had to go $(200 - 160)$ km = 40 km more and we have already seen that for 40 km it took 30 minutes. Then, it reached Asansol at 11 hr 15 min + 30 min i.e., at 11.45 a.m. and it left that station at 11.45 a.m. + 5 min or 11.50 a.m.

Note : Now calculate and verify whether the train reached Madhupur actually at 12.50 p.m.

Example 6. A train is 75 m long. If the train takes 20 seconds to cross a tree by the railway line, what is the speed of the train?

After the front part of the train just left the tree, its rear part reached the same spot in 20 seconds.

Hence, the distance covered by the train in 20 seconds is its length.

∴ In 20 seconds the train goes 75 m.

“ 1 “ “ “ “ $\frac{75}{20}$ m.

In 1 hr = 60×60 “ “ “ $\frac{75 \times 60 \times 60}{20}$ m

= $\frac{27}{2}$ km = 13.5 km.

∴ the speed of the train is 13.5 km/hr.

Example 7. A train, 100 metres long, crosses a platform, 120 metres long, in 40 seconds. What is the speed of the train?

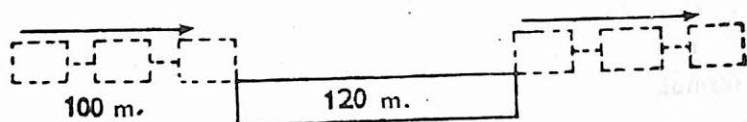


Fig. 3

A moving train starts crossing a platform when the front part of the train comes in line with one end of the platform (see the first position of the train in the figure) and the crossing ends when the rear part of the train just leaves the other end of the platform (see the second position of the train in the figure). Thus, to cross the platform, the train will have to travel a distance equal to the sum of its own length and the length of the platform. In the present case the train will have to travel $(100+120)$ m or 220 m.

$\therefore$ in 40 secs the train goes 220 metres
in 1 sec „ „ „ $\frac{220}{40}$ metres

In 1 hr = 60×60 secs. „ „ $\frac{220 \times 60 \times 60}{40 \times 1000}$ km.
= 19.8 km.

$\therefore$ the speed of the train is 19.8 km/hr.

Exercise 16

1. Find speed by unitary method and verify with the help of formula:

- (i) Distance = 40 km., time = 2 hrs.
- (ii) Distance = 250 cm., time = 5 mins.
- (iii) Distance = 75 m., time = 15 secs.

2. Find distance by unitary method and verify with the help of formula:

- (i) Speed = $37\frac{1}{2}$ km/hr., time = 6 hrs.
- (ii) Speed = 52.5 cm./sec., time = 5 hrs.
- (iii) Speed = 85 m./sec., time = $2\frac{1}{17}$ secs.

3. Find time by unitary method and verify with the help of formula:

- (i) Speed = 11 km./hr., distance = 550 km.
- (ii) Speed = 17 cm./sec., distance = 5 metres.
- (iii) Speed = $6\frac{1}{2}$ m./sec., distance = $19\frac{1}{2}$ metres.

4. A train starts from Hooghly at a speed of 80 km/hr. What time will it take to reach a station 120 km from Hooghly?

5. Kamalbabu left Balimore on a bicycle at 10.00 a.m. and reached Tribeni at 10.45 a.m. If the speed of the bicycle is 8 km/hr, what is the distance between Balimore and Tribeni?

6. A bus leaves Calcutta at 7.30 a.m. and reaches Digha at 12.00 noon. If the speed of the bus is 45 km/hr, what is the distance between Calcutta and Digha?

7. One man reaches bus-stand from his house in 3 minutes. The distance between the house and the bus-stand is 320 metres. Find out the speed of the man in kilometre per hour.

8. Mr. Mukherjee started from his house on a motor cycle at 8.00 a.m. to meet a relative in his house. After spending 1 hour at his relative's house he came back home at 11.30 a.m. If the speed of the motor cycle is 40 km/hr, what is the distance between his house and his relative's house?

9. A train, 75 metres long, travels at a speed of 60 km/hr. What time will the train take to cross a tree?

10. The speed of a train is 48 km/hr. The train crosses a platform, 120 metres long, in 15 secs. What is the length of the train?

11. In what time will a train, 85 metres long, moving at a speed of 45 km/hr cross a bridge 65 metres long?

12. The points A, B and C are on a straight line and the point B lies between A and C. Sudipta and Nilesch start at the same time from the points A and B respectively towards the point C. Nilesch walks at a speed of 3 km/hr. Sudipta reached C after $2\frac{1}{2}$ hrs on a motor cycle and found that Nilesch reached there

$\frac{1}{2}$ hr earlier. The distance between A and B is 120 km. What is the speed of the motor cycle?

13. The distance of Gopalpur from Ramnagar is 18 km. Harish starts from Ramnagar at a speed of 5 km/hr. He takes rest for 5 minutes after every 2 kms. If he starts at 6.30 a.m., when will he reach Gopalpur?

14. Two trains start from the same places at the same time and in the same direction at speeds of 50 km/hr and 60 km/hr. What will be the distance between them after 40 minutes? If the two trains start from the same place, at the same time and in opposite directions, what will be the distance between them after 40 minutes?

15. Two trains start at the same time towards each other from two stations 100 km apart. If the speeds of the trains are 60 km/hr and 75 km/hr, what will be the distance between them after 40 minutes and after $1\frac{1}{2}$ hrs?

16. A passenger train and a goods train start from the same station, at the same time and in the same direction. After 40 minutes it is found that the goods train is 20 km behind the passenger train. If the speed of the passenger train is 60 km/hr, what is the speed of the goods train?

17. A man goes 1200 metres in 45 minutes by a cycle-rickshaw. What time will he take to go 8 km. by the same rickshaw?

18. Diptenbabu takes 1 hr 20 min to reach Arambag in his car, but he takes 1 hr. to come back. If the speed of the car was 60 km/hr at the time of going, what was its speed at the time of coming back?

19. A ferry boat goes to the other side of the river at a speed of 6 km/hr in still water. It takes 5 minutes to do so. But at the time of coming back, because of wind in the opposite direction, the speed of the boat is reduced and the time taken is 8 minutes. What is the speed of the boat at the time of coming back?

20. In 220 metre sprint the boy finishing first reached the finishing line in 56 seconds and found that the second and the third boys were 4 metres and 6 metres back respectively. What times would be taken by the second and the third boys to reach the finishing line? What was the speed of each of the boys?

Chapter 6

Simple Interest

6.1. If we keep money at bank or post office and withdraw the same after some time, we get some extra money. This extra money is called *interest*. The amount of money that is deposited is called *principal* or *capital*. The sum of interest and principal is called the *amount*.

Rs 200 is deposited with a bank for 3 years. At the end of 3 years a simple interest of Rs 36 is received.

Here, principal or capital = Rs 200, interest = Rs 36.

Amount = Rs 200 + Rs 36 = Rs 236.

Now, on Rs 200 simple interest for 3 years = Rs 36

„ Re 1 „ „ „ 3 „ = Rs $\frac{36}{200}$

= 18 Paise

„ Re 1 „ „ „ 1 year = $\frac{18}{3}$ = 6 P

On Rs 100 simple interest for 1 year = Rs 6

The simple interest obtained by taking a definite amount as unit of money and a definite length of time as unit of time is called *rate of interest* or simply *rate*. Here, simple interest on Re 1 for 1 year 6 paise, we may say the rate of interest is 6 paise per rupee per year. By the help of unitary method we find that in this case, the simple interest per rupee per month is $\frac{6}{12}$ paise or $\frac{1}{2}$ paise and the simple interest on Rs 100 for 1 year is Rs 6.

Generally, the rate of interest is taken as interest on Rs 100 for one year and this is called annual rate of interest per cent or rate percent per annum. Thus, here the annual rate of interest percent is 6. We also express this rate of interest as '6% per annum'.

From the above example, we find that monthly $\frac{1}{2}$ paise per rupee, annually 6 paise per rupee and annually Rs 6 per Rs 100—all these convey same rate of interest in different units.

Likewise, rate of interest $7\frac{1}{2}\%$ per annum will mean that interest on Rs 100 for 1 year is Rs $7\frac{1}{2}$.

If money is kept in a bank for different periods of time, the rate of interest will be different. This rate may, again, be different at different points of time. The rates of interest may also be different in different banks.

If a man deposits some money with a bank, the bank gives him a pass book wherein the account of the deposited money is shown. On money deposited, at different times, the interest is added. The interest will be high if the deposit is high. The interest will also increase with the period for which money is kept. If some money is withdrawn, the total deposit decreases and so, in that case, the interest will be less. In the next two pages, the pictures of a pass-book used and inside page of the pass-book are given.

In calculating interest, we take 12 months = 1 year and generally 1 month = 30 days. But in

calculating interest for a period from a particular date of one month to a particular date of another month, we calculate total number of days by considering actual number of days in those months and subtract 1 day from the above total. In that case we take $1 \text{ yr} = 365 \text{ days}$.

From the above discussion, we get :

$$(1) \text{ Amount} = \text{Principal} + \text{Interest.}$$

$$\text{Principal} = \text{Amount} - \text{Interest.}$$

$$\text{Interest} = \text{Amount} - \text{Principal.}$$

If we know any two of the above, we can find out

Name Subhash Mandal

Account No. C-23 Ledger Folio C-1/27

XYZ BANK

SAVINGS BANK

PASS BOOK - 23

---CALCUTTA---BRANCH.

Fig. 4

decrease. If the principal is doubled or trebled, the simple interest will be doubled or trebled, again if the principal is made half or one-fifth or one-seventh, the simple interest will, likewise, decrease to half or one-fifth or one-seventh. We can arrive at these changes with the help of unitary method.

Similarly, keeping the principal unchanged if the time is increased by two, three, four, etc. times, the simple interest will also increase by two, three, four, etc. times. Again, if the time decreases and becomes half, one-third or one-fourth, the simple interest will also decrease and become half, one-third or one-fourth. These changes can also be found by unitary method.

Again, if these two types of changes occur together, then the result can also be found by unitary method. However, in these cases the unitary method has to be applied twice. Again, if the simple interest remains unchanged, the relationship between principal and time is of reverse nature. If same simple interest is earned from more money, the time will be less. Again, if same simple interest is earned in more time, the principal will be less. In such cases, if the principal is increased by a number of times, the time required will be decreased and equal to the original time divided by the above number. The application of unitary method in solving such problems is shown in the following examples.

Example 1. If the rate of simple interest is $7\frac{1}{2}\%$ per annum, what will be the interest on Rs 100 in 7 years? What will be the amount at the end of those seven years?

Note that, here the principal is unchanged, only the time is changing. We can solve this problem by applying unitary method once.

On Rs 100 simple interest for 1 year is Rs $7\frac{1}{2}$
 $= \text{Rs } \frac{15}{2}$

„ Rs 100 „ „ 7 years is Rs $\frac{15 \times 7}{2}$
 $= \text{Rs } \frac{105}{2} = \text{Rs } 52\frac{1}{2}$

$\therefore$ simple interest for 7 years is Rs $52\frac{1}{2}$
 $= \text{Rs } 52.50$

The amount at the end of that period will be
 Rs $(100 + 52.50) = \text{Rs } 152.50$.

Example 2. What will be the simple interest on Rs 800 at $6\frac{1}{4}\%$ per annum for $3\frac{1}{2}$ years? What will be the amount at the end of this period?

The rate of interest is $6\frac{1}{4}$ percent per annum.

$\therefore$ On Rs 100 the interest for 1 year is
 Rs $6\frac{1}{4} = \text{Rs } \frac{25}{4}$

„ Re 1 „ 1 year is Rs $\frac{25}{4 \times 100}$

„ Rs 800 „ „ 1 „ „ Rs $\frac{25 \times 800}{4 \times 100}$

[Here we have applied unitary method once]

On Rs 800 the interest for $3\frac{1}{2}$ years is

$$\text{Rs } \frac{25 \times 800 \times 7}{4 \times 100 \times 2} = \text{Rs } 175$$

[Here we have applied unitary method second time]

$\therefore$ required interest = Rs 175

$\therefore$ amount = Rs $(800 + 175) = \text{Rs } 975$

Hence, if the principal (P), rate of interest percent per annum (R) and the time (T) are given, the simple interest (I) will be as follows.

$$\text{Simple Interest} = \frac{\text{Principal} \times \text{Rate of interest per cent per annum} \times \text{time}}{100}$$

$$\text{or, } I = \frac{PRT}{100}$$

Exercise 17

Calculate simple interest: (1—6)

1. On Rs 500 for 1 year at 5% per annum.
2. On Rs 62.50 for 4 years at 10% per annum.
3. On Rs 4565 for 2 yrs. 6 months at 4% per annum.
4. On Rs 866.25 for 4 months at $7\frac{1}{8}\%$ per annum.
5. On Rs 146 for 1 day at $2\frac{1}{2}\%$ per annum.
6. On Rs 1000 for the period from 1st January to 27th May, 1982 at 6% per annum.
7. For house-repair Enayet took a loan of Rs 6000 from a bank at 12% per annum. After 5 years he refunded the loan along with the interest. How much did he return?
8. For the cultivation of his field Paltan Murmu took a loan of Rs 1600 from a co-operative bank for 2 years at 6% per annum. What amount would he have to refund at the end of 2 years?
9. Manik, a student of Class VII of a school, saved Rs 125 from his pocket money. He deposited that money with a bank at 5% per annum. What simple interest will he get after $2\frac{1}{2}$ years?
10. Bhombal and Najrul are two friends. To run a small business together they took a loan of Rs 15000 from a bank at $12\frac{1}{2}\%$ per annum. What simple interest will they have to pay after 8 years?
11. What will be the amount of Rs 400 for 8 years at $6\frac{1}{2}\%$ per annum?
12. Madhabbabu deposited with the post office a sum of

Rs 5000 for 7 years at 10% per annum. What amount will he get at the end of the period?

13. Mrs. Ghosh deposited in a fixed account with a bank a sum of Rs 550 for 4 years at $9\frac{1}{2}\%$ per annum. She kept the amount received after 4 years in the savings account of another bank for $2\frac{1}{2}$ years at $5\frac{1}{2}\%$ per annum. What will be the simple interest in the savings account after $2\frac{1}{2}$ years?

14. What will be the amount of Rs 500 at 5% per annum for 5 years?

15. Ikbal deposited with a bank a sum of Rs 200 at $5\frac{1}{2}\%$ per annum on 1st January. After 6 months he deposited another sum of Rs. 200 with the same bank at the same rate. What simple interest will he receive at the end of the year?

6.2. Finding Principal:

Let us see how to find out the principal when simple interest, time and rate percent per annum are given. If we are to find principal by general unitary method, it is necessary to keep principal on the rear right hand side of the first given statement. In this connection, we have to keep in mind that if the time and rate remain unchanged, principal will increase or decrease with the increase or decrease of simple interest. Similarly, we may say that if the simple interest and the rate percent per annum remain unchanged, principal will increase or decrease with increase or decrease of time.

See the following sums:

Example 1. A sum of money is deposited with a bank for 4 years at 6% per annum. At the end of the period the simple interest obtained is Rs 225. What is the principal?

• Rate of interest is 6% per annum.

When interest for 1 yr is Rs 6, the principal is Rs 100

$$\text{,, ,, ,, 1 ,, Re 1, ,, ,, Rs } \frac{100}{6}$$

$$\text{,, ,, ,, 1 ,, Rs 225, ,, ,, Rs } \frac{100 \times 225}{6}$$

$$\text{,, ,, ,, 4 yrs Rs 225, ,, ,, Rs } \frac{100 \times 225}{6 \times 4}$$

$$= \text{Rs } 937.50$$

[Note the last step. Here, the simple interest is unchanged, time is increased from 1 year to 4 years. Here, the principal will not be increased to 4 times, but will be reduced to one-fourth].

$$\therefore \text{ required principal} = \text{Rs } 937.50$$

Note that in this sum unitary method has been applied twice. Once the simple interest has been changed from Rs 6 to Re 1 and then it is changed to Rs 225. For this change via 1 (unit), this method is called unitary method. Second time the unitary method is applied to change the time from 1 year to 4 years. If for more time same simple interest is obtained, then the capital or principal will be less. Hence, in this case instead of multiplying by 4, we have divided by 4.

Example 2. What principal will amount to Rs 5160 at $7\frac{1}{2}\%$ in 8 years?

Simple interest on Rs 100 in 1 year is Rs $7\frac{1}{2} = \text{Rs } \frac{15}{2}$.

$$\text{,, ,, ,, 100 ,, 3 yrs ,, Rs } \left(\frac{15}{2} \times 8\right)$$

$$= \text{Rs } 60.$$

Hence, at $7\frac{1}{2}\%$ in 8 years Rs 100 will amount to
 $\text{Rs } 100 + \text{Rs } 60 = \text{Rs } 160$

Now, if the amount is Rs 160 the principal is Rs 100

„ „ „ „ Re 1 „ „ „ Rs $\frac{100}{160}$

„ „ „ „ Rs 5160 „ „ „ Rs $\frac{100 \times 5160}{160}$
= Rs 3225

∴ required principal = Rs 3225.

Here, we have first applied unitary method in finding the amount in 8 years when the principal is Rs 100. Then the principal is found by applying unitary method again. In this way, unitary method is applied twice.

Note: The relationship between principal and amount should be specially noted. If the rate of interest and time remain unchanged, the amount will increase or decrease by the number of times by which the principal increases or decreases. Hence, from the change of any one of them, the corresponding change in the other can be determined by unitary method. See the following example:

Example 3. Gurubhakta Rajbansi deposited Rs 700 with a bank at a certain rate for certain time and at the end of the period, it amounted to Rs 900. What sum should be deposited at the same rate and for same time so that the amount will be Rs 1350?

Amount is Rs 900 when principal is Rs 700

„ „ „ „ Re 1 „ „ „ Rs $\frac{700}{900}$

„ „ „ „ Rs 1350 „ „ „ Rs $\frac{700 \times 1350}{900}$
= Rs 1050.

Here, the principal could be directly found from the amount by unitary method because the rate at which one of principal and amount increases or decreases, the other also increases or decreases at the same rate. But the change of amount in relation to time and rate of interest is not at the same rate. That is why unitary method cannot be directly applied in this case. The change of simple interest part of the amount is, in this case, at the same rate of change of time or rate of interest. That is why, in sums of this type the simple interest is to be first found by unitary method and then the amount is to be calculated. Note the following example:

Example 4. What will be the amount of Rs 750 at $6\frac{1}{2}\%$ in 6 years? From the same principal if the amount has to be Rs 1200, what will be the rate of interest? If the rate of interest remains unchanged, in what time will the same principal amount to Rs 1140?

Simple interest on Rs 100 in 1 year is Rs $\frac{13}{2}$.

“ “ “ “ “ 6 “ “ Rs $(\frac{13}{2} \times 6)$
=Rs 39

In 6 years Rs 100 will amount to Rs $(100 + 39)$
=Rs 139

“ “ Rs 750 “ “ “ Rs $\frac{139 \times 750}{100}$
=Rs 1042.50

If Rs 750 amounts to Rs 1200, the simple interest will be Rs $(1200 - 750) = \text{Rs } 450$.

Simple interest on Rs 750 in 6 years is Rs 450

$$\text{Rs } \frac{450}{750 \times 6}$$

$$\text{Rs } \frac{450 \times 100}{750 \times 6} = \text{Rs } 10$$

$\therefore$ required rate is 10% per annum.

If the amount is Rs 1140, the simple interest will be Rs $(1140 - 750) = \text{Rs } 390$.

Again, if the rate of interest remains unchanged Rs $\frac{1}{2}$ is the simple interest on Rs 100 in 1 year

$$100 \text{ in } \left(1 \div \frac{13}{2}\right) \text{ yr} = \frac{2}{13} \text{ year}$$

$$1 \text{ in } \frac{2 \times 100}{13} \text{ years}$$

$$750 \text{ in } \frac{2 \times 100}{13 \times 750} \text{ years}$$

$$390 \text{ in } 750$$

$$\text{in } \frac{2 \times 100 \times 390}{13 \times 750} \text{ years} = 8 \text{ years.}$$

$\therefore$ required time is 8 years.

Note: First question of the above example is about relative changes of principal and amount. This change can be found by unitary method. Hence, we could get the answer in this case by directly applying unitary method. But the second question of

the example is about relative changes of amount and rate of interest. The characteristic of such change is that the simple interest part only of the amount can be found by unitary method. That is why, here the unitary method has been applied after separating the simple interest. Same is the case in the third question. The change of time with simple interest is under unitary method, not with amount. Here also the time has been found after separating the simple interest from the amount.

Exercise 18

Find principal (1—6):

1. If simple interest is Rs 126 in 5 years at 3% per annum.
2. If simple interest is Rs 10.50 in 2 yrs 4 months at $4\frac{1}{2}\%$ per annum.
3. If simple interest is Rs 32.50 in 6 months at $6\frac{1}{4}\%$ per annum.
4. If simple interest is Rs 2145 in 3 years 3 months at $7\frac{1}{2}\%$ per annum.
5. If simple interest is Re 1 in 1 day at 5% per annum.
6. If simple interest is Rs 31.20 for the period, from 1st January to 7th August of 1980, at 8% per annum.
7. A man deposits a sum of money with a bank at $5\frac{1}{4}\%$ per annum. After 2 years he receives Rs 84 as simple interest. What is his principal?
8. Radhanathbabu deposits a sum of money with a bank for 6 months at 5% per annum and after 6 months he receives Rs 13.20 as simple interest. What is his principal?
9. To start a poultry Gautam takes loan from a co-operative bank at 12% per annum. He pays Rs 37.80 as simple interest per month. What is the amount of his loan?
10. By depositing a sum of money with a bank at 10% per annum, Mrs. Suchitra Ghosh receives Re 1 as simple interest per day. What money did she deposit with the bank?

6.3. Finding time:

If principal, simple interest and rate of interest per annum are given, the time can be found by unitary method. In this case, from the given statement, time has to be written to the rear right. Relative increase or decrease in time with the change in principal, simple interest and rate of interest per annum has to be determined properly.

Example 1. Ajit Kapasia deposited Rs 12500 with a bank at $5\frac{1}{2}\%$ per annum. After a certain period of time, he received Rs 2750 as simple interest. Find the time for which Ajitbabu kept the money in the bank.

Rs 100 yields Rs $\frac{11}{2}$ (Rs $5\frac{1}{2}$) simple interest in 1 yr.

Rs 1 „ „ $\frac{11}{2}$ „ „ „ 1×100 years.

[If same simple interest is earned on less principal, the money should be kept in the bank for much more time. That is why instead of dividing by 100, we have multiplied by 100.]

Rs 1 yields Re 1 simple interest in $\frac{1 \times 100 \times 2}{11}$ years

[If the simple interest decreases the time will also decrease; hence, we have divided here.]

Rs 12500 yields Re 1 simple interest in $\frac{1 \times 100 \times 2}{11 \times 12500}$ yrs.

[More money will yield same simple interest in less time i.e., we will have to divide.]

Rs 12500 yields Rs 2750 simple interest in

$$\frac{100 \times 2 \times 2750}{11 \times 12500} \text{ years} = 4 \text{ years.}$$

[To earn more interest on same principal the time required will be more and hence we will have to multiply.]

$\therefore$ required time = 4 years.

Exercise 19

Find time: (1—6)

1. If simple interest on Rs 6400 at $4\frac{1}{2}\%$ per annum is Rs 1008.
2. If simple interest on Rs 675 at $7\frac{1}{2}\%$ per annum is Rs 202.50.
3. If simple interest on Rs 128 at $6\frac{1}{4}\%$ per annum is Rs 1.60.
4. If simple interest on Rs 500 at 5% per annum is Rs 50.
5. If simple interest on Rs 624 at $3\frac{1}{2}\%$ per annum is Rs 9.10.
6. If simple interest on Re 1 at 12% per annum is 1 paise.
7. Rs 650 deposited with a bank at 5% per annum for some time. If the simple interest is Rs 217.75, what is the time for which the money was kept in the bank?
8. What is the time in which Rs 625.50 will amount to Rs. 750.60 at $4\frac{1}{2}\%$ per annum?
9. A sum of Rs 75 is kept with the post office for some time at $8\frac{1}{2}\%$ per annum. After that time the amount is Rs 100. What is the time for which the money was kept?
10. The price of a wrist watch is Rs 300. A student deposits Rs 240 with a bank at 5% per annum. After what time will he get just sufficient money to be able to buy the wrist watch.
11. A lends Rs 3500 to B and Rs 2500 to C at 5% per annum. After how many years will he get a total simple interest of Rs 1200 from the two?
12. A certain principal is doubled in 20 years; in how many years will it be trebled?
13. In how many years will a certain sum of money be doubled at 6% per annum?
14. Gafoor took a loan of Rs 520 at 5% per annum and paid Rs 91 as simple interest at the time of repaying loan. After how many years of receiving the loan did Gafoor repay it?

6.4. Finding rate of interest per annum:

In the same manner as above we may find the rate of interest per annum when the simple interest, principal and time are given. In this case we are to start by writing the given statement keeping the simple interest to the rear right and then to find the interest on Rs 100 for 1 year from that statement. Proper attention should be given to the type of changes in principal and time with the change in simple interest. It should also be remembered that rate of interest 5% means rate percent of interest is 5 (not 5%).

Example 1. At what rate percent will the simple interest on Rs 5000 for 8 years 3 months be Rs 5500?

$$8 \text{ years } 3 \text{ months} = 8\frac{1}{4} \text{ years} = \frac{33}{4} \text{ years.}$$

$$\text{Simple interest on Rs 5000 for } \frac{33}{4} \text{ years} = \text{Rs } 5500$$

$$\text{,, ,, ,, Re 1 ,, } \frac{33}{4} \text{ ,,} = \text{Rs } \frac{5500}{5000}$$

[If principal decreases, simple interest will also decrease—that is why we are to divide.]

$$\text{Simple interest on Re 1 for 1 year} = \text{Rs } \frac{5500 \times 4}{5000 \times 33}$$

$$\text{,, ,, ,, Rs 100 ,, 1 ,,} = \text{Rs } \frac{5500 \times 4 \times 100}{5000 \times 33}$$

[If principal increases, simple interest will also increase; hence we are to multiply.]

$$= \text{Rs } \frac{40}{3} = \text{Rs } 13\frac{1}{3}.$$

$\therefore$ rate of interest per annum is $13\frac{1}{3}\%$.

Remember the following:

(1) If the rate of interest and time remain unchanged, the simple interest will increase or decrease with the increase or decrease of principal.

(2) If the rate of interest and principal remain unchanged, simple interest will increase or decrease with the increase or decrease of time.

(3) If the rate of interest and the simple interest remain unchanged, time will decrease when principal increases and time will increase when principal decreases.

(4) If the time and simple interest remain unchanged, principal will decrease when rate of interest increases and principal will increase when rate of interest decreases.

(5) If the time and principal remain unchanged, rate of interest will increase when simple interest increases and rate of interest will decrease when the simple interest decreases.

(6) If the simple interest and principal remain unchanged, the rate of interest will decrease if time increases and the rate of interest will increase if time decreases.

Increase and decrease referred to above have a special characteristic—the ratios of the corresponding increase or decrease will always be same. When two quantities are increasing simultaneously, if one is made two or three or four etc. times the other will also be two or three or four, etc. times and same is true when both are decreasing simultaneously. Again, if one decreases as a result of increase in another, when one increases by a

number of times the decreased value of the other will be obtained by dividing it by the above number—this is the relation between principal and time when the rate of interest and the simple interest remain unchanged.

Exercise 20

Find the rate of interest per annum: (1—6)

1. If simple interest on Rs 500 for 4 years is Rs 100.
 2. If simple interest on Rs 650 for 2 years is Rs 52.
 3. If simple interest on Rs 970 for 2 years 6 months is Rs 48·50.
 4. If simple interest on Rs 3157·50 for 4 months is Rs 105·25.
 5. If simple interest on Rs 7300 per day is Re 1.
 6. If simple interest on Rs 917·50 for the period from 1st March to 18th December is Rs 55·05.
 7. From a fixed deposit of Rs 5000 with a bank Fatehma Bibi earns Rs 750 annually. What is the rate of interest per annum?
 8. From a fixed deposit of Rs 1500 with a bank Dhanraj Tamang earns Rs 75 after 6 months. What is the rate of interest per annum?
 9. Madhu Bag took a loan of Rs 1000 for digging a pond. He has to pay Rs 3·75 as simple interest per month. What is the rate of interest per annum?
 10. One bank pays Rs 50 as simple interest on Rs 500 for $3\frac{1}{2}$ years; another bank pays Rs 50 as simple interest on Rs 350 for 5 years. Find the rate of interest per annum of the two banks.
 11. After 6 years of taking a loan Mannan Mian found that the simple interest was $\frac{3}{8}$ of the principal. What was the rate of interest per annum?
-

Chapter 7

Area

7.1. You have often noticed that if a solid is kept on a plane, in many occasions, the solid occupies some portion of the plane. As for example, if you keep a book on the bench in your class room; the book will occupy some portion of the top plane of the bench; again if you keep an inkpot instead that will also occupy some portion—but less than that occupied by the book. Obviously some portion of the plane will be occupied only if the surface of the solid in contact of the plane is also plane.

[Note: Plane figures may be of different shapes and sizes.]

Main subject of discussion of this chapter is how to measure these small or large figures that can be marked on a plane? We face this question quite often.

Notice how you face this question at home or school.

Suppose, in school you have to draw a figure of geometry on one part of the black board and to work out a sum of arithmetic on the other part. You may have a small piece of land by the side of your house. You may like to put some flowering plants on that land. Your father may ask you to do so on one part of the land and to put pepper saplings on the other part.

Here also the question is to measure small or large portions of plane surface. Again suppose

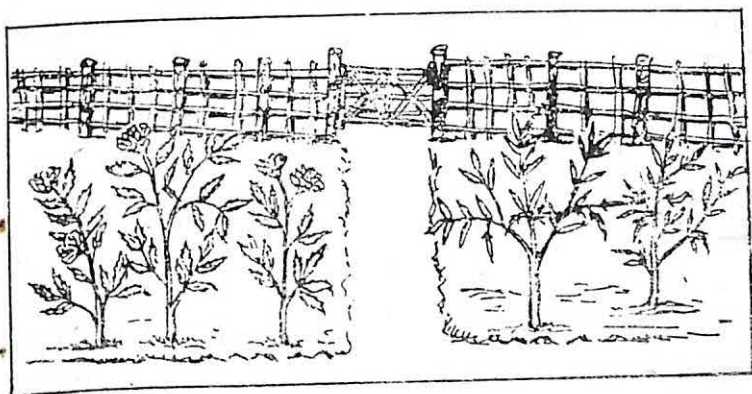


Fig. 6

your father and the father of your classmate Ajij have adjacent lands for cultivation. Whose land is more and by how much—all these are questions of measuring plane figures. Thus in our everyday life we cannot avoid this question. Hence in arithmetic also we shall discuss this question in this chapter.

The measurement of a certain portion of plane surface is called its area. But how to measure this portion? To measure anything we need a unit—as for example for measuring length we have taken metre or centimetre as unit. This metre or centimetre is nothing but a certain length. Similarly, for measuring weight we have taken gram or kilogram as unit. These are also certain weights. In the same manner we have to take a unit for measuring area.

We shall take the area of a square of length 1 cm and breadth 1 cm (as in fig. 7) as a unit of area and shall call it 1 square centimetre or in short 1 sq. cm. or 1 cm.^2 .

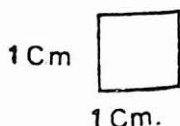


Fig. 7

At present we shall discuss only about the area of a rectangle. You will learn how to find out areas of other plane figures later on. Incidentally it may be mentioned that you are already familiar with rectangle.

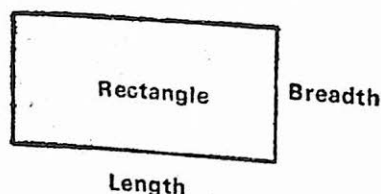


Fig. 8

Opposite sides of a rectangle are equal and parallel.

Square is a special type of rectangle; all the sides of a square are equal.

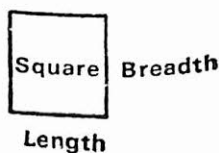


Fig. 9

Four rectangles are given in figures 10—13. Their lengths and breadths are different. We divide each one of them into some squares of area 1 sq. cm. by drawing lines parallel to length and breadth. The length of first rectangle is 2 cm. and its breadth is also 2 cm. Count and see that its area is 4 sq. cm. Again the length and breadth of the second rectangle is 5 cm. and 3 cm. By counting we find that its area is 15 sq. cm.

In like manner find the length, breadth and

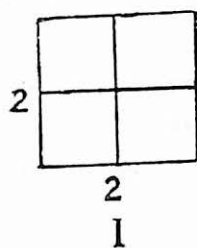


Fig. 10

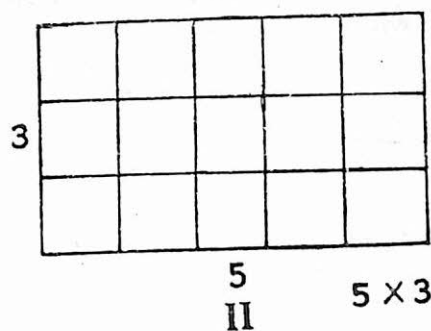


Fig. 11

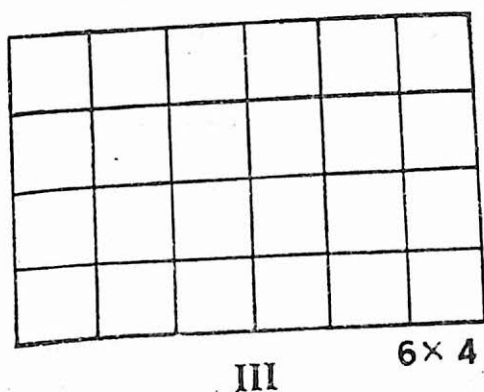


Fig. 12

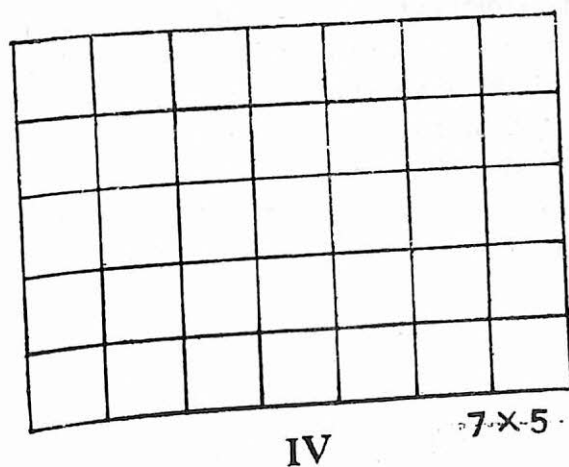


Fig. 13

area of the other two rectangles and complete the following table:

Fig. No.	Length cm.	Breadth cm.	Area sq.cm.	Length $\times$ Breadth sq.cm.	Conclusion
10	2	2	4	$2 \times 2 = 4$	Area of rectangle = length $\times$ breadth sq. cm.
11	5	3	15	$5 \times 3 = 15$	
12	6	4			
13	7	5			

Thus we find that the area of each rectangle is equal to the product of its length and breadth. Hence, generally, we can say,

$$\text{Area of a rectangle} = \text{length} \times \text{breadth (sq. units)} \quad \dots (i)$$

We should remember that the unit of length or breadth is different from the unit of area. Unit of measuring length of an object is cm., metre, etc. But area denotes the portion of a plane which is occupied by that object and the unit of its measurement is sq. cm., sq. m. etc. By 1 sq. m. we mean the portion of plane occupied by a square of length 1 m. and breadth 1 m.

$$\begin{aligned} \text{Length and breadth of a square are same, and so} \\ \text{area of a square} &= \text{length} \times \text{length (sq. unit)} \\ &= (\text{length})^2 \text{ (sq. unit)} \quad \dots (ii) \end{aligned}$$

$\therefore$ product of length and breadth is area of rectangle

$$\therefore \text{length} = \frac{\text{area of rectangle}}{\text{breadth}} \quad \dots \text{ (iii)}$$

$$\text{breadth} = \frac{\text{area of rectangle}}{\text{length}} \quad \dots \text{ (iv)}$$

$$\therefore (\text{length})^2 = \text{area of square}$$

$$\therefore \text{length} = \text{square root of area of square} \quad \dots \text{ (v)}$$

The sum of all four sides of a rectangle or square is called its perimeter.

$$\text{Perimeter of a rectangle} = 2 \times (\text{length} + \text{breadth}) \quad \dots \text{ (vi)}$$

$$\text{Similarly, perimeter of a square} = 4 \times \text{length} \quad \dots \text{ (vii)}$$

If the length, breadth and height of a room are given, the area of its four walls

$$= 2 \times \text{length} \times \text{height} + 2 \times \text{breadth} \times \text{height}$$

$$= 2 \times (\text{length} + \text{breadth}) \times \text{height}$$

$$= \text{Perimeter} \times \text{height}.$$

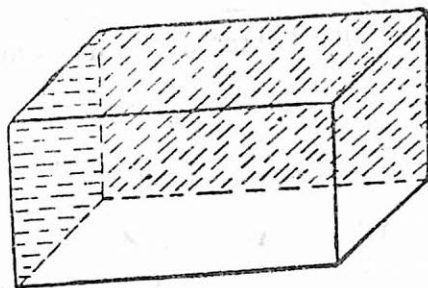


Fig. 14

[We are to remember that there are two walls along the length and two walls along the breadth and the height of each wall is same.]

The results obtained so far may be listed as follows:

Given	Required measurement = ?
Length and breadth of a rectangle	$\text{Area} = \text{length} \times \text{breadth}$
„	$\text{Perimeter} = 2 (\text{length} + \text{breadth}).$
Area and breadth of a rectangle	$\text{Length} = \text{area} \div \text{breadth}$
Area and length of a rectangle	$\text{Breadth} = \text{area} \div \text{length}$
Any one side of a square	$\text{Area} = \text{side} \times \text{side} = (\text{side})^2$
„	$\text{Perimeter} = 4 \times \text{side}$
Area of a square	$\text{Length of a side} = \text{square root of area}$
Length and breadth of a floor	$\text{Area of floor} = \text{length} \times \text{breadth}$
„	$\text{Area of roof (or ceiling)} = \text{length} \times \text{breadth}$
Length, breadth and height of a room	$\text{Area of four walls}$ $= 2(\text{length} + \text{breadth}) \times \text{height}$ $= \text{perimeter} \times \text{height}$

Example 1. The length and breadth of a rectangular garden are 150 m. and 80 m. What is its area? What will be the cost of putting grass in the garden

at the rate of 5 Paise per square metre? What is the perimeter of the garden? What will be the cost of fencing the garden at the rate of 80 Paise per metre?

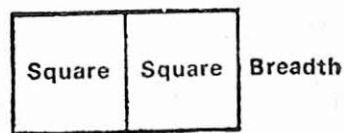
$$\begin{aligned}\text{Area of the garden} &= \text{length} \times \text{breadth} \\ &= 150\text{m.} \times 80\text{ m.} = 12000\text{ sq.m.}\end{aligned}$$

$$\text{Cost of putting grass} = \text{Rs } \frac{12000 \times 5}{100} = \text{Rs } 600.$$

$$\begin{aligned}\text{Perimeter of the garden} &= 2 (\text{length} + \text{breadth}) \\ &= 2(150 + 80)\text{ m.} \\ &= 2 \times 230\text{ m.} = 460\text{ m.}\end{aligned}$$

$$\text{Cost of fencing} = \text{Rs } \frac{460 \times 80}{100} = \text{Rs } 368.$$

Example 2. The length of a rectangular plot is double its breadth. Its area is 578 sq. m. What are the length, breadth and perimeter of the plot?



Length = 2 X Breadth

Fig. 15

Here we see the figure of the given plot. We divide the plot into two equal parts by drawing a line parallel to the breadth from the midpoint of the length. Since the length is double the breadth, by the above division we get two equal square plots from the given rectangular plot.

$$\begin{aligned}\text{Area of each square} &= (578 \div 2)\text{ sq.m.} \\ &= 289\text{ sq. m.} = 17^2\text{ sq.m.}\end{aligned}$$

We know that any one side of a square
= square root of its area.

Hence the length of any one side of the square
 $= \text{square root of } 289 \text{ sq.m.} = 17 \text{ m.}$

This is the breadth of the given rectangular plot.

Length of the rectangular plot $= (17 \times 2) \text{ m.} = 34 \text{ m.}$

Perimeter $= 2(34 + 17) \text{ m.} = (2 \times 51) \text{ m.} = 102 \text{ m.}$

Example 3. How many square tiles 25 cm. long will be needed to cover a courtyard, 20 m. long and 15 m. broad?

Area of the courtyard $= 20 \text{ m.} \times 15 \text{ m.} = 300 \text{ sq.m.}$

Area of one tile $= 25 \text{ cm.} \times 25 \text{ cm.}$

$$= 625 \text{ sq. cm.} = \frac{625}{10000} \text{ sq. m.}$$

$\therefore$ required number of tiles

$$\begin{aligned} &= \frac{\text{Area of the courtyard}}{\text{Area of one tile}} \\ &= \frac{300 \times 10000}{625} = 4800 \end{aligned}$$

[Note: 1 sq.m. $= 1 \text{ m.} \times 1 \text{ m.}$

$= 100 \text{ cm.} \times 100 \text{ cm.}$

$= 10000 \text{ sq. cm.}$

$\therefore 625 \text{ sq. cm.} = \frac{625}{10000} \text{ sq. m.}]$

Example 4. Length, breadth and height of a room are 5 m., 4 m. and 3 m. respectively. The room has one door 2 m. $\times$ 1.2 m and two windows 1.25 m. $\times$ 1.2 m.

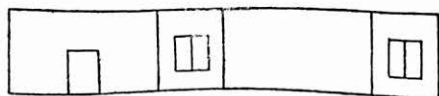
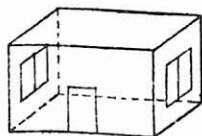


Fig. 16

(i) What is the area of floor of the room? What will be the cost of cementing the floor at a rate of Rs 25 per square metre?

(ii) What is the area of four walls of the room without the door and windows?

(iii) What is the area of the ceiling?

(iv) What will be the cost of white washing the inside walls, without the door and windows, and the ceiling of the room at Re .85 per square metre?

Answer: (i) Floor area of the room

$$= \text{length} \times \text{breadth}$$

$$= (5 \times 4) \text{ sq. m.} = 20 \text{ sq. m.}$$

Cost of cementing the floor at Rs 25 per square metre = Rs (20×25) = Rs 500.

(ii) Area of door = (2×1.2) sq. m. = 2.4 sq. m.

Area of two windows = $2 \times (1.2 \times 1.25)$ sq. m.
= 3 sq. m.

Inside area of the four walls along with door and windows = $2 \times (\text{length} + \text{breadth}) \times \text{height}$

$$= 2 \times (5 + 4) \times 3 \text{ sq. m.} = 54 \text{ sq.m.}$$

Inside area of the four walls without the door and windows

= Total area of the four walls

— Area of door — Area of two windows.

$$= 54 \text{ sq. m.} - 2.4 \text{ sq.m.} - 3 \text{ sq. m.}$$

$$= 54 \text{ sq. m.} - 5.4 \text{ sq. m.} = 48.6 \text{ sq. m.}$$

(iii) Area of ceiling = length $\times$ breadth

$$= (5 \times 4) \text{ sq.m.} = 20 \text{ sq.m.}$$

(iv) Inside area of the four walls without the door and the windows along with the ceiling

$$= 48.6 \text{ sq. m.} + 20 \text{ sq.m.} = 68.6 \text{ sq. m.}$$

Cost of white washing the walls and the ceiling
 $= \text{Rs } (68.6 \times .85) = \text{Rs } 58.31$
 $= \text{Rs } 58 \text{ and } 31 \text{ Paise.}$

Example 5. Length and breadth of a garden are 35 m. and 25 m. respectively. There is a path, 1.5 m. broad, all around the outside of the garden.
 (i) What is the area of the path? (ii) What will be the area of the path if it were inside the garden?

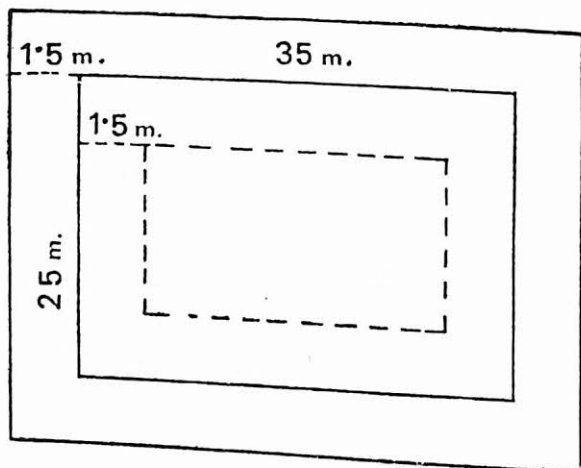


Fig. 17

(i) Length of the garden along with outside path
 $= (35 + 2 \times 1.5) \text{ m.} = (35 + 3) \text{ m.} = 38 \text{ m.}$

Breadth of the garden along with outside path
 $= (25 + 2 \times 1.5) \text{ m.} = (25 + 3) \text{ m.} = 28 \text{ m.}$

$\therefore$ Area of the garden along with outside path
 $= 38 \text{ m.} \times 28 \text{ m.} = 1064 \text{ sq. m.}$

Area of the garden $= (35 \times 25) \text{ sq. m.} = 875 \text{ sq. m.}$

Hence, area of the outside path

$$= 1064 \text{ sq. m.} - 875 \text{ sq. m.} = 189 \text{ sq. m.}$$

(ii) Length of the garden without the inside path
 $= 35\text{m.} - 2 \times 1.5\text{m.} = 32\text{m.}$

Breadth of the garden without the inside path
 $= 25\text{m.} - 2 \times 1.5\text{m.} = 22\text{m.}$

Area of the garden without the inside path
 $= (32 \times 22) \text{ sq. m.} = 704 \text{ sq.m.}$

$\therefore$ Area of the inside path
 $= \text{Area of the garden} - \text{area of the garden without the inside path}$
 $= 875 \text{ sq.m.} - 704 \text{ sq.m.} = 171 \text{ sq.m.}$

Exercise 21

1. (i) What is the area of a rectangle of length 45 cm. and breadth 35 cm.?
- (ii) What is the length of a rectangle of area 550 sq. cm. and breadth 22 cm.?
- (iii) What is area of a square, the length of one of its sides being 22 cm.?
- (iv) What is the breadth of a rectangle of length 24 cm. and area 360 sq.cm.?
- (v) Find the length of one side of a square whose area is 729 sq. cm.
- (vi) What is the perimeter of a square of side 4 cm.?
- (vii) What are the length of a side and area of a square whose perimeter is 20 cm.?

(viii) What is the perimeter of a rectangle of length 15 cm. and breadth 12 cm.?

(ix) What is the perimeter of a square of area 81 sq. cm.?

(x) What is the area of the four walls of a room $5\text{m.} \times 4\frac{1}{2}\text{m.} \times 3\frac{1}{2}\text{m.}$?

(xi) What is the total area of 3 windows of $1\cdot3\text{m.} \times 1\cdot25\text{ m.}$?

(xii) What is the area of a path 1 m. broad along the outside of a square of length 20 m.?

(xiii) Around outside of a rectangle of length 45 m. and breadth 20 m., there is a path 1.5 m. broad. What will be the cost of metalling the path at Rs 1.20 per square metre? How much less would have been the cost had the path been around inside of the rectangle?

2. (a) Fill in the blanks of the following table by putting correct values with appropriate units.

Name of figure	Given	Required
Rectangle	length = 5m. breadth = 3m.	area =
	area = 30 sq. cm. length = 6 cm.	breadth =
	area = 3.6 sq.m. breadth = 1.2 m.	length =
Square	length of one side = 3 mm.	area =
	length of one side = 5 m	perimeter =
	area = 64 sq.m.	length =

(b) Followings are the figures of some plots of land. Their lengths and breadths are also given in the figures. Find the areas of the shaded portions of the figures.

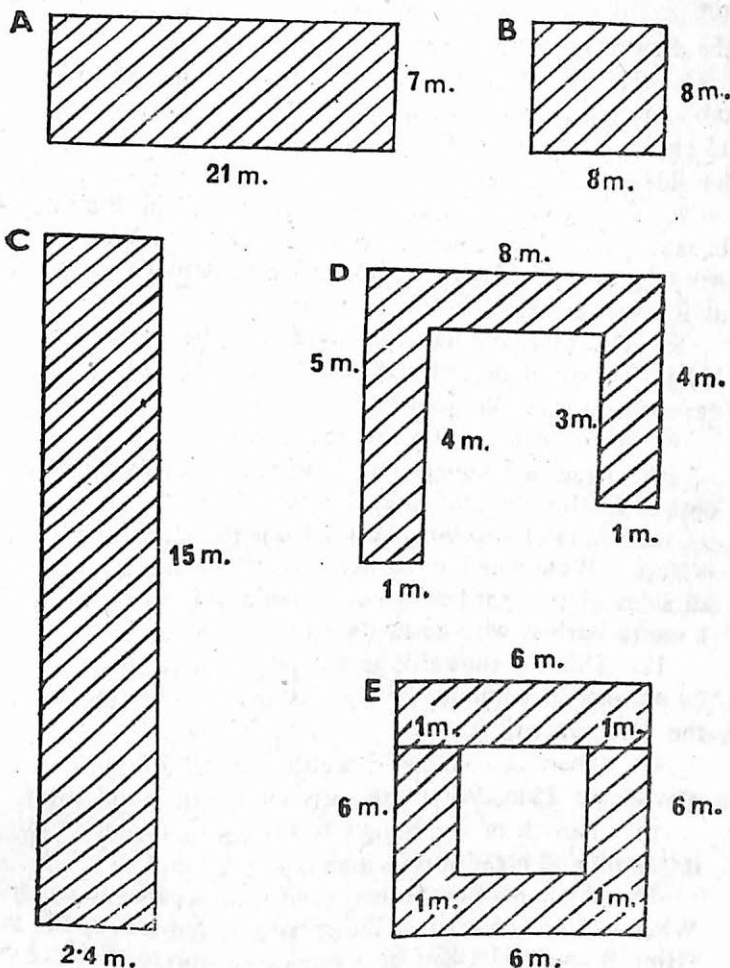


Fig. 18

3. The length and breadth of your room are 5 m. and 3.8 m. respectively. On the floor there is a carpet $3\frac{1}{2}$ m. $\times$ $1\frac{1}{4}$ m. What is the area of the floor outside the carpet?

4. A drawing paper is 1.2 m. long and 1 m. broad. What is the total area of the two sides of the paper?
5. A page of your exercise book is 15 cm. long and 12 cm. broad. The page is full of writing having 2 cm. margin all around. What is the area of the written portion of the page? What is the area of the blank portion of the page?
6. The length and breadth of the rectangular top of your table are 1.8 m. and 1 m. respectively. If each of your book is 15 cm long and 10 cm broad, how many books can be kept side by side on the table?
7. One room of a school at Raigunge is 6 m. long and 4 m. broad. How many tiles of length 50 cm. and breadth 40 cm. will be required to cover the floor? What will be the total cost at Rs 4.50 per tile?
8. The floor of a hall $10\text{m.} \times 8\text{m.}$ is to be covered by carpet. If 1 sq. m. carpet costs Rs 18, what will be the total cost for fully carpeting the whole floor?
9. The perimeter of a square plot is 64 m.; what is its area?
10. Area of a square plot is 6400 sq. m. What will be the cost of fencing the plot at Rs 1.50 per metre?
11. Area of a rectangular garden is 800 sq. m. and its breadth is 20 m. What length of barbed wire will be necessary for fencing all sides of the garden once? What will be the total cost if 1 metre barbed wire costs Rs 3.12?
12. One boy runs around a pond of length 30 m. and breadth 20 m. once in 1 minute. What distance will he run in 1 hour at the same speed?
13. There is a path of breadth 1 m. all around outside of a plot $30\text{m.} \times 25\text{m.}$ What is the perimeter of the plot with the path?
14. Length of a rectangle is 3 times its breadth. What are its length and breadth if its area is 432 sq. m.?
15. 4 cm. sq. portion is cut out from a paper $16\text{cm.} \times 12\text{cm.}$ What will be the area of the remaining portion of the paper? (Hint: $4\text{cm.}^2 = (4 \times 4)\text{sq. cm.} = \text{area of a square of side 4 cm.}$)
16. $1\frac{1}{2}\text{m.} \times 1\frac{1}{2}\text{m.}$ portion is cut out from a wooden plank $3\frac{3}{4}\text{m.} \times 2\frac{1}{2}\text{m.}$ What is the area of the remaining portion of the plank?
17. A rectangular cardboard is 2.5 m. long and 1.5 m. broad. 4 pieces of 10 cm. sq. are cut out of that card board. What is the area of the remaining portion of the cardboard?

18. Each bunch of paddy plant requires $50 \text{ cm.} \times 50 \text{ cm.}$ land. How many such bunches can be planted in a field $45 \text{ m.} \times 40 \text{ m.}$?

19. Each student needs at least $1.5 \text{ m.} \times 1.2 \text{ m.}$ floor space. The measurement of your class room is $6 \text{ m.} \times 4.8 \text{ m.}$. What is the maximum number of students that can be accommodated in that room?

20. Each of the three saris of Kamala is $5 \text{ m.} \times 1.15 \text{ m.}$. What is the total area of cloth of the three saris?

21. Latif's garden is 10 m. long and 8 m. broad. If 5 sq. m. land is required for each rose sapling, how many saplings can be planted in the garden?

22. A cultivator has 1600 sq. m. of land. In that land he has a dwelling house of length 10 m. and breadth 6 m. and a pond 18 m. long and 15 m. broad. The rest of the land is utilised for vegetable garden. What is the area of the vegetable garden?

23. There is $2\frac{1}{2} \text{ m.}$ broad boundary all around a pond 25 m. long and 22 m. broad. What is the area of the boundary of the pond?

24. A garden is 35 m. long and 25 m. broad. All around inside of it there is a path $2\frac{1}{2} \text{ m.}$ broad. (i) What is the area of the path? (ii) If the path would have been outside the garden, what would have been its area?

25. The length, breadth and height of a room are 6 m. , 5 m. and 3 m. respectively.

(i) What is the area of its four walls?

(ii) If the room has one $2\frac{1}{2} \text{ m.} \times 2 \text{ m.}$ door and three $1\frac{1}{2} \text{ m.} \times 1 \text{ m.}$ windows, what will be the area of the four walls without the door and windows?

(iii) What will be the total cost of cementing the floor of that room at Rs 9.50 per square metre?

(iv) What will be the total cost of white washing the four walls, without the door and the windows, and the ceiling at 50 Paise per square metre?

26. What will be the cost of preparing a road of length 10 km. and breadth 3 m. at Rs 2.50 per square metre?

27. The 'Pradhan' of Saptagram sanctioned Rs 570 as cash and 380 kg of wheat at Rs 1.50 per kg. for making an earthen road 4 m. broad and 95 m. long. What is the expenditure per square metre of the road?

Answers

Exercise 1

1. No, Yes 2. 2249280, 2 3. (i) 1, 3, 5, 7, 15, 21, 35, 105 (ii) 3, 5 or 3, 7 or 5, 7 (iii) 1, 3, 5, 15; 15, as this is largest among the common factors. 4. (i) 42, 84, 126, 168, 210, 252, 294, 336, 378, 420 (ii) 105, 210, 315, 420, 525, 630, 735, 840, 945, 1050 (iii) 210, 420; 210, as this is smallest among the multiples. 5. (i) 99937, 10001 (ii) yes. 6. 15 7. 852 8. 180, 360 9. 17 m. 50cm; 5, 7 10. 24; 3, 4, 6, 8, 12.

Exercise 2

1. (i) $5 \div 2$, $5 - 2$, $5 + 2$, 5×2 , 5^2 (ii) $\frac{6}{11}$, $\frac{5}{7}$, $\frac{3}{4}$, (iii) $\frac{8}{9}$, $1\frac{1}{2}$, $1\frac{2}{3}$ 2. (i) $\frac{7}{11}$ (ii) $1\frac{7}{11}$ (iii) $\frac{7}{10}$ (iv) $1\frac{1}{8}$ (v) $1\frac{1}{8}$ (vi) $39\frac{1}{2}$ (vii) $\frac{3}{4}$ (viii) $\frac{4}{7}$ (ix) 1 (x) 1 (xi) $6\frac{5}{8}$ (xii) $1\frac{5}{8}$ 3. $91\frac{1}{8}$ kg. 4. Rs 1280 5. $\frac{1}{8}$, $\frac{4}{85}$, $\frac{1}{10}$

Name	Expenditure			Savings
	Food	Clothing	Others	
Raicharan	Rs 200	Rs 50	Rs 100	Rs 50
Haricharan	Rs 150	Rs 20	Rs 140	Rs 40
Brindaban	Rs 342	Rs 76	Rs 95	Rs 57

6. 600, Maya—500, Chhaya—480 7. Rs 12000, Rs 3000, Rs 2000 8. 40.

Exercise 3

1. (i) .077, .079, .69, .7 (ii) $\frac{2}{5}$, .75, $\frac{39}{50}$, .781 2. (i) 9461.29 (ii) 2.83211 (iii) .0106704 (iv) 10200000 (v) .026 (vi) 7.4 3. .248, .06875, .2142857 4. 2880 km., 1920 km. 5. 4 kg, 2kg, 1kg, $\frac{1}{2}$ kg. 6. Mustak—28 years, Hanif—32 years 7. Rs 800, Rs 400, Rs 20 8. .004375 cm 9. 2.04 m. 10. 650 km 11. 17 times, 1.08 litres 12. Rs 2, Rs 2.40, Rs 1.20 13. 10m 14. Rs 50000.

Exercise 4

1. (i) 24 (ii) 32 (iii) 39 2. 21 3. 340, Rs 3.40
4. 6 5. 3.27 6. 900 7. 68, 17 8. 10 9. 42 10. 24, 16.

Exercise 5

1. 15 days 2. 8 days 3. 32 4. Rs 2200 5. Rs 2.26
6. Rs 183 7. 441 8. Rs 1050 9. Loss Rs 40 10. 7h.

Exercise 6

1. (i) $\frac{13}{88}$, $\frac{4}{6}$, $\frac{7}{100}$, $1\frac{1}{8}$, $2\frac{1}{2}$ (ii) $\frac{11}{1000}$, $\frac{8}{125}$, $\frac{32}{125}$, $\frac{17}{2000}$, $260\frac{1}{100}$
(iii) $253\frac{1}{8}$, 9, $\frac{1}{6}$, 10120, 25025 2. (i) .32 (ii) .175 (iii) 1.625
(iv) 2.34 (v) 1.5625 (vi) 4.367 3. (i) Rs 3375 (ii) 15.25kg.
(iii) Re 1 (iv) 562.5 km. (v) Rs 40000 (vi) 45 crores 50 lacs.

Exercise 7

1. (a) 3.8, 5.5, 8.8 (b) 3, 5, 8 (c) 9 (d) 8, 8, 8
2. (a) $12\frac{1}{2}$, $35\frac{3}{5}$, $78\frac{7}{8}$ (b) 12, 35, 78 (c) 99 (d) $1\frac{1}{8}$,
 $\frac{3}{8}$, $\frac{7}{8}$ 3. (a) $3\frac{3}{8}$, $16\frac{6}{8}$, $124\frac{4}{4}$ (b) .8, 1.6, $12\frac{4}{4}$ (c) 3.0,
16.1, 124.12 (d) 90 (e) $\frac{3-0}{100}$, $\frac{16-1}{100}$, $\frac{124-12}{100}$ 4. (a) $\frac{1}{8}$,
 $\frac{1}{16}$, $\frac{3}{16}$, $1\frac{101}{888}$ (b) $\frac{1}{80}$, $\frac{1}{6}$, $\frac{7}{100}$, $\frac{701}{3888}$, $\frac{1061}{8888}$ (c) $\frac{103}{878}$, $\frac{11066}{11066}$,
 $\frac{5401}{24078}$, $\frac{11376}{11376}$ (d) $\frac{1801}{1801}$ or $\frac{1301}{1000}$, $\frac{3084}{999}$ or $31\frac{25}{99}$, $\frac{787}{988}$ or $21\frac{21}{88}$,
 $\frac{101101}{99900}$ or $1\frac{1201}{8888}$ (e) $\frac{3479}{888}$ or $7\frac{14}{100}$, $\frac{58}{88}$ or $1\frac{4}{88}$, $\frac{308}{99}$ or $2\frac{1}{10}$,
 $\frac{1482007}{200000}$ or $29\frac{12158}{100000}$ (f) $\frac{84}{88}$, $\frac{528}{2275}$, $\frac{422}{99}$ or $4\frac{22}{99}$, $\frac{120158}{100000}$ or
 $4\frac{1158}{200000}$.

Exercise 8

1. 5, 459, 45904, 459040 2. Andhra Pradesh—279, 27910,
279097; Assam—44, 4431, 44314; Bihar—108, 10847, 108474;
Karnatak—119, 11940, 119395; Tamilnadu—246, 24634, 246345
3. 45, 46; 45.9, 45.97, 45.974; 46.0, 45.97, 45.975 4. 45, 45,
45.3, 45.3, 45.30, 45.31, 45.306, 45.307; 4, 4, 4.3, 4.4, 4.36, 4.36,
4.364, 4.365; 1, 1, 1.0, 1.0, 1.00, 1.00, 1.000, 1.001; 0, 1, .6,
.7, .67, .67, .673, .674; 8, 9, 8.5, 8.5, 8.54, 8.54, 8.543, 8.544;
0, 0, 0, .1, .06, .06, .061, .062; 0, 0, 0, 0, 0, 0, .001, .001
5. (i) .2, (ii) 3.14 (iii) .1 (iv) 104.667 6. .62 mile
7. 299330 km 8. .4 inch 9. 57.27 degrees 10. £5.51
11. (i) Rs 1.83 (ii) 28 paise (iii) Rs 21.42, Rs 28.56,
No, 2 Paise more.

Exercise 10

1. (i) $\frac{2}{5}, \frac{1}{5}, \frac{3}{10}, \frac{1}{10}, \frac{2}{25}; \frac{7}{10}, \frac{7}{50}, \frac{7}{30}, \frac{7}{40}, \frac{7}{60}$
 (ii) $3\frac{1}{2}, 1\frac{3}{4}, 1\frac{1}{8}, \frac{7}{8}, \frac{7}{10}; 2\frac{1}{2}, 1\frac{1}{10}, \frac{17}{24}, \frac{17}{32}, \frac{17}{40}$
 (iii) $\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \frac{4}{40}, \frac{5}{50}; \frac{7}{10}, \frac{7}{20}, \frac{7}{30}, \frac{7}{40}, \frac{7}{50}$
 (iv) $\frac{7}{8}, \frac{7}{16}, \frac{7}{24}, \frac{7}{32}, \frac{7}{40}; 1\frac{5}{16}, \frac{21}{32}, \frac{7}{16}, \frac{21}{80}, \frac{21}{80}$
 (v) $2\frac{5}{6}, 1\frac{2}{3}, \frac{18}{10}, \frac{4}{5}, \frac{18}{10}; 5\frac{5}{6}, 2\frac{3}{4}, 1\frac{11}{12}, 1\frac{3}{4}, 1\frac{7}{8}$
 (vi) $1\frac{2}{3}, \frac{5}{6}, \frac{5}{6}, 1\frac{5}{6}, \frac{1}{3}; 2\frac{1}{2}, 1\frac{1}{4}, \frac{5}{8}, \frac{5}{8}, \frac{1}{2}$
 (i) no, (ii) no, (iii) no, (iv) yes, $\frac{7}{10}$, (v) no, (vi) yes, $\frac{5}{8}$.
2. (i) $\frac{17}{36}, \frac{17}{72}, \frac{17}{108}, \frac{17}{144}, \frac{17}{180}, \frac{17}{216}$ (ii) $\frac{7}{36}, \frac{7}{72}, \frac{7}{108}, \frac{7}{144}, \frac{7}{180}, \frac{7}{216}$ (iii) $\frac{1}{18}, \frac{1}{27}, \frac{1}{36}, \frac{1}{45}, \frac{1}{54}, \frac{1}{63}$ (iv) $\frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \frac{1}{25}, \frac{1}{30}, \frac{1}{36}$ (v) $\frac{2}{3}, \frac{2}{6}, \frac{2}{9}, \frac{2}{12}, \frac{2}{15}, \frac{2}{18}$ 3. (i) $\frac{1}{8}$
 (ii) $\frac{5}{8}$ (iii) $\frac{2}{27}$ (iv) $\frac{2}{27}$ (v) $\frac{1}{36}$ (vi) $\frac{1}{36}$ (vii) $\frac{1}{108}$ (viii) $\frac{1}{216}$.

Exercise 11

1. (i) $\frac{3}{4}, 1\frac{1}{2}, 2\frac{1}{4}, 3$ (ii) $\frac{5}{7}, 1\frac{3}{7}, 2\frac{4}{7}, 2\frac{6}{7}$ (iii) $2\frac{2}{3}, 5\frac{1}{3}, 8, 10\frac{2}{3}$ (iv) $\frac{9}{8}, 1\frac{1}{8}, 2\frac{1}{8}, 3$ (v) $\frac{5}{8}, \frac{5}{8}, \frac{5}{8}, 1\frac{1}{8}$ (vi) $\frac{1}{2}, 1\frac{1}{2}, 1\frac{3}{2}, 2\frac{1}{2}$ 3. (i) 4, 8, 12 (ii) $4\frac{1}{2}, 8\frac{1}{2}, 12\frac{1}{2}$ (iii) $6\frac{1}{2}, 13\frac{1}{2}, 20\frac{1}{2}$ (iv) 42, 84, 126 (v) $46\frac{1}{2}, 93\frac{1}{2}, 140\frac{1}{2}$ (vi) 60, 120, 180 (vii) 21, 42, 63 (viii) $1\frac{1}{2}, 2\frac{1}{2}, 4\frac{1}{2}$.

Exercise 12

1. (i) $2\frac{1}{2}$ (ii) $1\frac{1}{2}$ (iii) $3\frac{1}{2}$ (iv) $2\frac{3}{8}$ (v) $7\frac{1}{2}$ (vi) $38\frac{1}{2}$
 (vii) $7\frac{1}{2}$ (viii) $27\frac{3}{8}$ (ix) 42 (x) 60 (xi) 60 (xii) 90
 (xiii) $3\frac{3}{4}$ (xiv) $52\frac{1}{2}$ (xv) $94\frac{3}{4}$ (xvi) 1386.

Exercise 13

1. (i) 6.4, 1.6 (ii) 4.6, .23 (iii) 6.6, 1.1 (iv) 8, .008 (v) 88.2
 .014 (vi) .8, .002 (vii) 262.5, .025 (viii) 8.1, .00001.

Exercise 14

1. (i) $21, \frac{1}{20}$; equal (ii) $7\frac{1}{2}, \frac{1}{48}$; equal (iii) $5\frac{1}{4}, \frac{7}{24}$; equal (iv) $48\frac{3}{4}, \frac{1}{100}$; equal (v) $1\frac{1}{2}, \frac{4}{3}$; equal 2. (i) $4\frac{1}{5}, \frac{1}{7}$; not equal (ii) $24, \frac{4}{100}$; not equal (iii) $2\frac{2}{11}, \frac{2}{8}$; not equal (iv) $56\frac{1}{4}, \frac{8}{100}$; not equal (v) $30, \frac{1}{100}$; equal 3. (i) 2 (ii) $3\frac{1}{2}$ (iii) $2\frac{1}{2}$ 4. (i) $\frac{5}{24}$ (ii) $\frac{5}{8}$ (iii) $\frac{1}{2}$ (iv) .016 (v) .003
 5. $\frac{4}{5}$ 6. $\frac{3}{10}$ 7. $2\frac{2}{3}$ 8. .15 9. 28.8 10. (a) (i) $31\frac{1}{2}$ (ii) $\frac{3}{10}$
 (iii) $\frac{2}{5}, \frac{1}{10}$ (iv) $3\frac{1}{2}$ (v) $\frac{2}{5}$ (b) (i) $4\frac{1}{2}, 2\frac{1}{10}; 31\frac{1}{2}; \frac{3}{10}; \frac{2}{5}; \frac{1}{10}; 3\frac{1}{2}; \frac{2}{5}$ (ii) H.C.F., L.C.M., L.C.M., H.C.F.

Exercise 15

1. (a) 2, 3, 5, 6 (b) (i) 9, 1 (ii) 3, 7 (iii) 3, 7 2. (i) 27 (ii) 31 (iii) 23 (iv) 25 (v) 39 (vi) 81 (vii) 36 (viii) 41 (ix) 252 (x) 825 (xi) 1002 (xii) 6930 (xiii) 10003 (xiv) 13765 (xv) 600098 3. 503, 200 4. 59, 136 5. 105 6. 64 7. 84, Rs 1.68 8. 9801, 1024 9. 100489 10. 32, 64.

Exercise 16

1. (i) 20 km/hr (ii) 50 cm/minute (iii) 5m/sec.
 2. (i) 225 km (ii) 11.250 km (iii) 175 m 3. (i) 50 hrs
 (ii) $29\frac{1}{17}$ sec. (iii) 3 sec. 4. $1\frac{1}{2}$ hr. 5. 6 km.
 6. 202.500 km 7. 6.4 km/hr 8. 50 km 9. 4.5 sec. 10. 80m
 11. 12 sec 12. 50.4 km/hr. 13. 10.46 a.m. 14. $6\frac{2}{3}$ km, $73\frac{1}{3}$ km
 15. 10 km, 102.500 km 16. 30 km per hour 17. 5 hr
 18. 80 km/hr 19. $3\frac{3}{4}$ km/hr 20. $57\frac{1}{8}$ sec., $57\frac{81}{107}$ sec.;
 $14\frac{1}{7}$ km/hr, $13\frac{1}{8}$ km/hr, $13\frac{3}{8}$ km/hr.

Exercise 17

1. Rs 25 2. Rs 25 3. Rs 456.50 4. Rs 21.18 5. 1 paise
 6. Rs 24 7. Rs 9600 8. Rs 1792 9. Rs 15.63 10. Rs 15000
 11. Rs 608 12. Rs 8500 13. Rs 104.36 14. Rs 625
 15. Rs 16.50

Exercise 18

1. Rs 840 2. Rs 100 3. Rs 1040 4. Rs 8800 5. Rs 7300
 6. Rs 650 7. Rs 800 8. Rs 528 9. Rs 3780 10. Rs 3650.

Exercise 19

1. $3\frac{1}{2}$ years 2. 4 years 3. 73 days 4. 2 years 5. 5 months
 6. 1 month 7. $6\frac{7}{10}$ years 8. $4\frac{1}{2}$ years 9. 4 years
 10. 5 years 11. 4 years 12. 40 years 13. $16\frac{2}{3}$ years
 14. $3\frac{1}{2}$ years.

Exercise 20

1. 5 2. 4 3. 2 4. 10 5. 5 6. $7\frac{1}{2}$ 7. 15 8. 10 9. $4\frac{1}{2}$
 10. $2\frac{1}{4}$, $2\frac{3}{4}$ 11. $6\frac{1}{2}\%$.

Exercise 21

1. (i) 1575 sq. cm. (ii) 25 cm. (iii) 484 sq. cm. (iv) 15 cm.
(v) 27 cm. (vi) 16 cm. (vii) 5 cm, 25 sq. cm. (viii) 54 cm.
(ix) 36 cm. (x) 66.5 sq. m. (xi) 4.875 sq. m. (xii) 84 sq. m.
(xiii) Rs 244.80, Rs 21.60 2. (a) 15 sq. m., 5 cm, 3m, 9 sq. mm,
20 m, 8m (b) (i) 147 sq. m. (ii) 64 sq. m. (iii) 36 sq. m.
(iv) 15 sq. m. (v) 20 sq. m. 3. 14.625 sq. m. 4. 2.4 sq. m.
5. 88 sq. cm, 92 sq. cm. 6. 120 7. 120, Rs 540 8. Rs 1440
9. 256 sq. m. 10. Rs 480 11. 120 m, Rs 374.40 12. 6 km 13. 118m
14. 36m, 12m 15. 176 sq. cm 16. 6.875 sq. m. 17. 3.71 sq. m
18. 7200 bunches 19. 16 20. 17.25 sq. m. 21. 16 22. 1270
sq. m. 23. 260 sq. m. 24. 275 sq. m, 325 sq. m. 25. (i) 66 sq. m.
(ii) 56.5 sq. m. (iii) Rs 285 (iv) Rs 43.25 26. Rs 75000 27. Rs 3.
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ALGEBRA



Chapter 1

Use of Symbol

1.1 You know that we use numbers for counting, we also express measures by numbers. But the numbers which are used in arithmetic have fixed values. They always represent definite measures. As for examples, you go to the market with Rs 10 and spend Rs 7, then you will be left with Rs $(10-7)$ or Rs 3. But in reality there are quite a number of things whose values are either not definitely known or always changing. In those cases it is not possible to express mathematical relationships by arithmetical number of definite value. Suppose you go to witness a game and ask your friend about the number of spectators. Your friend replies that the number would be two thousand. This number 'two thousand' is his guess. Actual number of spectators may be much above or below 2000. Similarly, if we say that Nihar has some money, we can not express the amount by a definite number, as he can have any amount of money. If you think a little bit you can have many such examples.

Because of this, apart from the numbers of arithmetic we also use a kind of indefinite numbers. We generally express these indefinite numbers by different English alphabates, for example, a, b, c, x, y, z , etc. The advantage of using number in this

way is that according to necessity they can represent any number. For example, in the first example discussed above we may suppose that the number of spectators in the field is x . Taking the number of spectators as x we can have all mathematical computations. But one thing should be remembered that x has no definite value.

If the number of spectators is 2000, the value of x will be 2000;

If the number of spectators is 2500, the value of x will be 2500, etc.

Similarly, in the second example you may suppose that Nihar has Rs a . Here also ' a ' may have any value. For example,

If $a=10$, Nihar will have Rs 10;

If $a=20$, Nihar will have Rs 20; etc.

From the above discussion we find that the letters x , a have been used to denote some numbers; but they may have different values. Thus the letters have been used here as symbols of numbers.

In algebra we can express any unknown number by symbol. For example, you do not know the length of the playing field of your school but you can suppose that the length is x metre. Karim's age is not known, we may assume that his age is y years.

Likewise we may assume,

Population of a village or town $= x$;

distance of Delhi from Calcutta $= z$ km;

Kamal's age $= y$ years; etc.

We can not only express unknown or indefinite numbers by using symbols but also use them in

mathematical computations. Now we would discuss this aspect.

When we use symbols instead of numbers, they actually mean some numbers, though the values of those numbers may not be definite. Hence, the rules of addition, subtraction, multiplication and division will be applicable to these symbols between themselves or in conjunction with arithmetical numbers. But the results that are obtained by applying the above rules cannot be expressed in the form found in arithmetic.

For example,

$$(1) \quad 2 \text{ mangoes} + 5 \text{ mangoes} = 7 \text{ mangoes.}$$

But if we take 5 mangoes with x mangoes, total number of mangoes will be $x+5$.

Note that we can write $2+5=7$, but we cannot simplify $x+5$ further. The reason for this is easily understandable. Here x has no definite value, it may have any value. Thus if we add 5 with x the result cannot be definite and accordingly, we write the result as $x+5$ using '+' sign. Similarly,

$$a \text{ metres} + 10 \text{ metres} = (a+10) \text{ metres};$$

$$\text{Rs } 10 + \text{Rs } b = \text{Rs } (10+b);$$

$$\text{Rs } x + \text{Rs } y = \text{Rs } (x+y); \text{ etc.}$$

(2) In case of subtraction also the result is expressed by using '-' sign. For example, Ruma's present age is 12 years, x years before her age was $(12-x)$ years. Similarly, if we spend Rs 5 from Rs y , we will be left with Rs $(y-5)$; if y eggs out of x eggs are rotten, the number of fresh eggs will be $(x-y)$.

(3) In case of multiplication the result is written differently in algebra.

In arithmetic we write

$$4 \times 5 = 20.$$

But in algebra,

$$4 \times x = 4x.$$

Similarly $a \times b = ab,$

$$m \times n = mn, \text{ etc.}$$

That is, the two numbers which are multiplied together are written side by side to express their product. Sometimes $4 \times x$ is also written as $4.x$.

Notes: (i) In writing the product of an arithmetical number with an algebraic number, which is denoted by English letters, we first write the arithmetical number followed by the alphabet; for example,

$$10 \times y = 10y,$$

$$y \times 10 = 10y \text{ and not } y10.$$

(ii) Product of a and b may be written as $a \times b$ or $a.b$ or ab , but we will make more use of the form ab .

(4) In case of division the quotient is expressed either by ' $\div$ ' sign or by fraction. For example, when a is divided by x the quotient will be

$$a \div x = \frac{a}{x}$$

Here dividend $= a$ and divisor $= x$.

Here dividend and divisor may have any value, but the divisor can never be zero.

Example 1. Express with the help of signs:—

- (i) 10 less than a .
- (ii) 5 more than b .
- (iii) 3 times x .
- (iv) one-fourth of y .
- (v) 2 more than 3 times m .
- (vi) 6 less than 10 times z .
- (vii) y less than x .
- (viii) Two times the product of x and y .
- (ix) Value of x is less than 12.
- (x) Value of x is greater than that of y .

Ans: (i) $a-10$, (ii) $b+5$, (iii) $3x$, (iv) $\frac{y}{4}$,
 (v) $3m+2$, (vi) $10z-6$, (vii) $x-y$, (viii) $2xy$,
 (ix) $x < 12$, (x) $x > y$.

Example 2. Write in words:—

- (i) $a-b$
- (ii) $m+n$
- (iii) $2a$
- (iv) $\frac{a}{3}$
- (v) $3p+2$
- (vi) $p+q+r$
- (vii) $\frac{x}{3}-2$.

Ans: (i) b less than a , (ii) sum of m and n ,
 (iii) two times a , (iv) one-third of a , (v) 2 more
 than 3 times p , (vi) sum of p , q and r , (vii) 2 less
 than one-third of x .

Example 3. You have 10 marbles, your friend

gives you x marbles more. How many marbles have you now?

Required number of marbles $= 10 + x$.

Example 4. You go to the market with Rs 10 and spend Rs x . How many rupees have you now?

If you spend Rs x from Rs 10, you will have Rs x less; that is you will have to subtract Rs x from Rs 10.

$\therefore$ the remainder $= \text{Rs } (10 - x)$.

Example 5. The sum of two numbers is a . If one of them is 10, what is the other?

The other number can be obtained by subtracting the given number from the given sum.

$\therefore$ the other number $= a - 10$.

Example 6. If a is greater than b , what will be the sum and difference of a and b ?

Sum $= a + b$

Difference $= a - b$.

Example 7. There are 10 benches in a class. If x students sit on one bench, how many students are there in the class?

To find the number of students, 10 should be multiplied by x .

Hence, the number of students $= 10 \times x = 10x$.

Example 8. You know that to write a number of two digits we are to multiply the digit in the ten's place by 10 and add with it the digit in the unit's place. For example, $45 = 4 \times 10 + 5$. Thus,

in a two-digit number if the digit in the ten's place is x and that in the unit's place is y , what will be the number?

The required number $= 10 \times x + y = 10x + y$.

Example 9. The length of the playing field of your school is 3 times the breadth. If the field is a metres long, what will be its breadth?

Here a metres have to be divided by 3. Hence, the field is $(a \div 3)$ metres or $\frac{a}{3}$ metres broad.

Exercise 1

1. Write by using signs :

- (i) sum of x and y , (ii) result of subtracting b from a ,
 (iii) k less than 10, (iv) sum of 5 and r , (v) 15 times x ,
 (vi) 5 less than two times x , (vii) one-seventh of b , (viii) 10
 less than the sum of x and y , (ix) value of z is less than 10,
 (x) value of x is equal to that of y .

2. Write in words :

- (i) $p+q$, (ii) $z-y$, (iii) $5a$, (iv) $\frac{x}{6}$, (v) $x+y+z$,
 (vi) $2p+r$, (vii) $m+2n$, (viii) $a-3b$,
 (ix) $\frac{p}{3} + 6$, (x) $\frac{a+2b}{3}$.

3. You have 5 exercise books, your father buys x more exercise books for you. How many exercise books will you have now?

4. Rahim had 15 marbles and he lost x marbles. How many marbles has he now?

5. Amulyababu is 60 years old. (i) What was his age x years before? (ii) What will be his age y years hence?

6. Mitali is 3 years older than Tumpa. If Tumpa is x years old, what is the age of Mitali?

7. If 1 dozen banana cost Rs x , what is the price of 4 dozen bananas?
8. Sum of two numbers is x . If one of them is 10, what is the other?
9. Difference of two numbers is y . If the larger number is 18, what is the smaller?
10. The product of two numbers is 64. If one number is p , what is the other?
11. Starting from home a man walks x km and then he goes y km by bus. What is the total distance that he has travelled?
12. On the Republic Day each student of a school is given two oranges. If the number of students in the school is x , find the total number of oranges that were given. If each orange costs 20 paise, what will be the total cost of the oranges?
13. A basket contained 200 oranges, out of which x oranges were rotten and the rest were divided equally among y boys and girls. Find the share of each.
14. If the price of 1 kg of rice is Rs x , what will be the price of 15 kg of rice? What quantity of rice can be bought for Rs 25?
15. (i) a km = how many metres?
(ii) y metres = how many kilometres?
16. The distance of Madhupur from Calcutta is z km. If one drives at 40 km per hour, what time will he take to go from Calcutta to Madhupur?
17. You are now x years old. What was your age 5 years before? What will be your age 15 years hence?
18. There are 10 rows of trees in a garden. If there are x trees in each row, what is the total number of trees in the garden?
19. One club has 50 members. If each member contributes x rupees, what will be the total contribution? With that money y blankets are purchased for flood-relief. What is the price of each blanket?
20. If the digit in the unit's place is x and that in the ten's place is y in a number of two digits, what is the number?
21. Mitu is x years older than Rinku. If Mitu's present age is y years, what is the age of Rinku? What will be the age of each 5 years hence?

1.2 Number and Expression :

You know that each of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 represent a number. You also know how to write other numbers using these symbols. We use numbers in counting something, for example, 10 men, 15 exercise books, 12 flowers, etc. Numbers are used not only in counting but also to denote measure. First, a certain quantity or amount of a measure is taken as 'unit'. Then the number of times the unit of the same class goes into the thing to be measured is a number which gives the measure of the thing. For example, a definite length is taken as 1 metre, that is metre is a unit of length. Likewise, 1 kilogram is a unit of weight, 1 hour is a unit of time, etc. If it is said that a rope is 10 m. long, we realise that the unit 1 metre goes into the length of the rope 10 times or the length of the rope is 10 times the 1 metre unit.

Similarly, 5×1 kilogram = 5 kilograms,

4×1 hour = 4 hours, etc.

Thus you see that to denote measure unit is attached to a number. This type of number with unit is called an expression. 10 metres, 5 kilograms, 3 hours, Rs 100, etc. are examples of different expressions. The number of times a unit goes into one expression is called its numerical value. For example, 10 is the numerical value of length of 10 metres and metre is its unit.

Algebraic expressions: If some numbers representing symbols and numbers are connected by

'+', '-', '×', '÷', etc. signs then the result is called algebraic expression or, in short, expression.

(i) $2a+3b+4c$,

(ii) $3a \times 4c \div 2$,

(iii) $a-b+c-d$, etc. are examples of expressions.

The portions of expression which are connected by '+', '-' signs are called terms. For example, in the expression $2a+3b-4c$ each of $2a$, $3b$, $4c$ is a term.

Expression having only one term is called monomial; for example, $2a$, $3b$, $8c$, etc. Expression having two terms is called binomial; for example, $2a+3b$, $4a-5c$ etc.

Similarly, expression having three or more terms is called a polynomial; for example, $a+b+c$, $2a-3b+4c+5d$ etc.

Coefficient: Any symbol or number connected with an expression as a multiplier is called the coefficient of the expression. Generally, the coefficient is written to the left of the expression. The coefficient of xy in $2xy$ is 2; again, the coefficient of y is $2x$ and that of x is $2y$.

Similarly, the coefficient of y in $4y$ is 4,
the coefficient of m in $3m$ is 3, etc.

Note : In an expression, if there is no number as multiplier, the coefficient is 1; for example, the coefficient of x is 1.

1.3 Use of brackets:

You have already come across different types of brackets like '—', (), { }, []. In algebra the expression inside same brackets is taken as one term. $2(a+b)$ means an expression which can be obtained by multiplying the sum of a and b by 2. But $2a+b$ means the sum of two times a and b .

$2x-\overline{3y-4x}$ and $2x-(3y-4x)$ will mean same expression. You will learn more in this respect afterwards.

Example 1. If $a=4$, $b=1$, $c=3$ and $h=2$, find values of the following expressions:

$$(i) \quad 3a-9b+c$$

$$(ii) \quad 6h+8b-5a.$$

$$\text{Ans: } (i) \quad 3a=3 \times a=3 \times 4=12$$

$$9b=9 \times b=9 \times 1=9$$

$$\text{and } c=3$$

$$\therefore 3a-9b+c=12-9+3=15-9=6.$$

$$(ii) \quad 6h=6 \times h=6 \times 2=12$$

$$8b=8 \times b=8 \times 1=8$$

$$5a=5 \times a=5 \times 4=20$$

$$\therefore 6h+8b-5a=12+8-20=20-20=0.$$

Example 2. If $a=5$, $b=3$, $c=1$, find values of the following expressions:

$$(i) \quad 2a-3(b-c)$$

$$(ii) \quad (a+b)-(b-c)$$

$$\text{Ans: } (i) \quad 2a=2 \times 5=10.$$

$$3(b-c)=3(3-1)=3 \times 2=6.$$

$$\therefore 2a-3(b-c)=10-6=4.$$

$$(ii) \quad a+b=5+3=8$$

$$b-c=3-1=2$$

$$\therefore (a+b)-(b-c)=8-2=6.$$

Example 3. If $a=2$, $b=1$ and $c=2$, find value of $2a \div 4b \times 5c$.

$$2a=2 \times 2=4, \quad 4b=4 \times 1=4, \quad 5c=5 \times 2=10.$$

$$\therefore \text{given expression} = 4 \div 4 \times 10 = 1 \times 10 = 10.$$

Exercise 2

If $a=2$, $b=5$, $c=1$ and $d=8$, find the values of the following expressions:

1. $a+b+c+d$.
2. $a \times b + c$.
3. $a \div c \times b$.
4. $b \div ac$.
5. $6+a-c+d$.
6. $2a+4c-b$.
7. $3a+5b+4c$.
8. $b \div a \div c$.
9. $6a \div b$.
10. $5a \times 3b \times 8c$.
11. $5d \div 4a \times b + 14c$.
12. $4ab+6d$.
13. $6ab+2bc+3ca+d$.
14. $ad-bc-ca$.
15. $\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$.
16. $\frac{2a+3b}{3b+4c}$.
17. $\frac{abc+bca+cab}{ab+bc+ca}$.
18. $\frac{2a+5b}{5c} + \frac{3d-8a}{a+b+c}$.
19. $5 \times \frac{b-c}{a} + 4 \times \frac{b-a}{d} + 3 \times \frac{d-a}{c}$.
20. The result of subtracting $(b+c+d)$ from 25.
21. Sum of a and b multiplied by d .

Chapter 2

Whole Number

2.1 Positive and Negative Whole Numbers :

You have definitely noticed how a carpenter fixes screw in a piece of wood. He fixes the screw-driver in the groove on the head of the screw and turns it in a certain direction. As a result of this the screw gradually enters into the wood. You have definitely noticed also that at the time of taking the screw out of the wood, the screw-driver is turned in a direction opposite to the direction in which it was turned for fixing the screw.

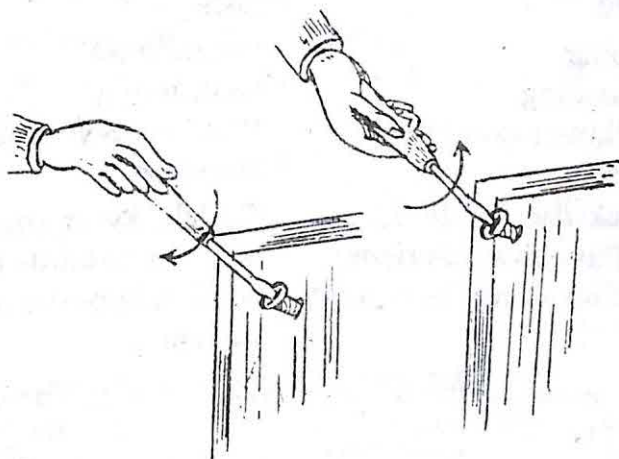


Fig. 1

If the screw-driver is to be turned 6 times at the time of fixing it, then it is to be turned 6 times

also at the time of fully unfixing it, but in the opposite direction. Though it is necessary to turn same number of times in both the cases, the nature of the turns is different. To denote methods of the same type and of opposite nature, as described above, one is taken as positive (with '+' sign) and the other is taken as negative (with '-' sign). For example, if the method of fixing the screw is taken as positive then that of unfixing it will be negative. Again, the method of fixing the screw is taken as negative, unfixing will be positive. Then, if we write 6 turns at the time of fixing as '+6', the 6 turns at the time of unfixing will have to be written as '-6'.

Followings are some quantities of the same class but of opposite nature:

Profit

Income

Ascending

Walking towards east

Increase

Clockwise rotation

Walking towards right

Motion along the current

Loss

Expenditure

Descending

Walking towards west

Decrease

Anticlockwise rotation

Walking towards left

Motion opposite to the current.

If we take one of these with '+' sign, the other will have '-' sign.

Examples : 1. If we denote ascending 5 metres by +5 metres, descending 3 metres will be -3 metres.

2. If we write movement of 2 km. towards east

as -2 km, movement of 1 km. towards west will be denoted by $+1$ km.

3. If a profit of Rs 50 is denoted by $+Rs\ 50$, a loss of Rs 20 will be denoted by $-Rs\ 20$.

The numbers discussed above have not only some numerical value, but there is a notion of direction attached to all of them. For example, a profit of Rs 15 or a loss of Rs 10 cannot simply be denoted by 15 or 10. The word 'profit' means a certain direction along which the quantity of money increases and the word 'loss' indicates a decrease in the quantity of money. If we indicate increase by '+' sign, the decrease will be indicated by '-' sign. Thus when a profit of Rs 15 is indicated by $+Rs\ 15$, a loss of Rs 15 will be indicated by $-Rs\ 15$. Number of this type which has not only a numerical value but also a direction is called a **directed number**.

We can very easily get the idea of directed number by geometric figure.



Fig. 2

Draw a straight line and take any point O on it. The straight line is extended indefinitely in both directions and the point O divides the straight line into two parts. One part is extended indefinitely to the right and the other to the left. Any one of these two directions, right and left, may be taken as

positive, then the other will be negative. Suppose the right direction is taken as positive then the left will be negative. Now taking a definite length as unit mark equal portions of the straight line both to the right and to the left starting from the point O. The first point of division to the right is situated at a distance 1 unit from the point O; we denote this point by $+1$. In this way, 2nd, 3rd, 4th, ...etc. points of division to the right will denote the numbers $+2, +3, +4, \dots$ etc. respectively.

Since the straight line is extended indefinitely to the right, all the points denoting 1, 2, 3, 4, ...etc. will be to the right of the point O. As these numbers are situated in the positive side, they are called **positive numbers**. In the same manner 1st, 2nd, 3rd, 4th, ...etc. points of division to the left of O will denote the numbers $-1, -2, -3, -4, \dots$ etc. Since the portion of the straight line to the left of O is also extended indefinitely, all the points denoting $-1, -2, -3, -4, \dots$ etc. will be on this portion to the left of O. As these numbers are situated in the negative side, they are called **negative numbers**. The point O denotes neither positive nor negative direction; this point denotes zero (0).

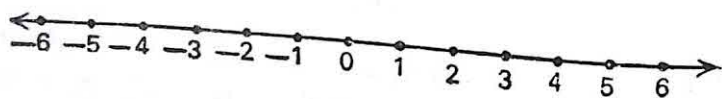


Fig. 3

Thus, we find that the numbers 1, 2, 3, 4, 5, ..., 0, and $-1, -2, -3, -4, -5, \dots$ etc. may be represented by the points on a straight line. These

numbers are called whole numbers. We can call this straight line, number-axis. The numbers 1, 2, 3, 4, 5, ... are called positive whole numbers and $-1, -2, -3, -4, -5, \dots$ are called negative whole numbers. Zero (0) is such a whole number which is neither positive nor negative.

You have already seen that the direction of increase is generally taken as positive and direction of decrease is taken as negative. Thus all the numbers situated to the right of O are greater than zero and all those to the left of it are less than zero. For this reason positive numbers are expressed by the sign ' >0 ' and the negative numbers by ' <0 '. Thus if x is a positive number we write $x > 0$, and if y is a negative number we write $y < 0$.

Note another point. From any point on the number-axis the more you move to the right, the value of the number will go on increasing and the more you go to the left, the value of the number will go on decreasing.

Thus, 1 is greater than 0, but -1 , is less than 0.

Using sign, we may write: $1 > 0$, but $-1 < 0$.

Similarly, $5 < 10$, but $-5 > -10$.

Natural numbers: For counting we use the numbers 1, 2, 3, 4, 5, ... etc. These numbers are called natural numbers.

We put '+' sign before natural numbers to denote positive numbers and '-' sign before them to denote negative numbers.

$+1, +2, +3, +4, \dots$ etc. are positive numbers and $-1, -2, -3, -4, \dots$ etc. are negative numbers.

Opposite numbers: If we watch the positive and negative numbers closely, we find that both of them contain natural numbers, but in case of positive numbers they have '+' sign and in case of negative numbers '-' sign. For example, the natural number in both of $+5$ and -5 is 5. Such numbers are called opposite of each other.

Opposite of $+5$ is -5 ,
and opposite of -5 is $+5$.

Similarly, $+3$ and -3 are opposite of each other
 $+10$ and -10 " " " " "

In general, we may say that $+a$ and $-a$ are opposite of each other.

Thus we find that every number other than 0 has an opposite (Fig. 4).

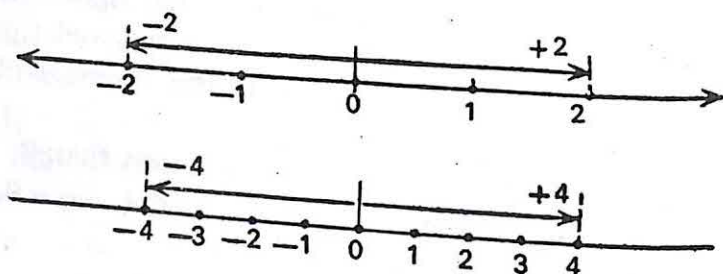


Fig. 4

Note a special characteristic of two mutually opposite numbers.

$+2$ is a positive number and it is situated at a distance of two units to the right of the point O. Now let us see what happens if -2 , the opposite of $+2$, is added to $+2$. Adding -2 with $+2$ means starting from the point $+2$ you have to go

2 units towards left (Fig. 5). Now see that as a result of this you have reached the point O or zero.

$$\text{Thus, } (+2) + (-2) = 0$$

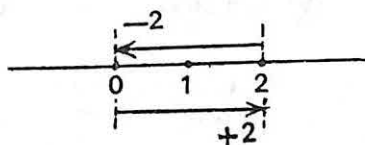


Fig. 5

In like manner test on the number-axis to verify that,

$$(+5) + (-5) = 0,$$

$$(+10) + (-10) = 0, \text{ etc.}$$

In general,

$$(+x) + (-x) = 0$$

that is, the sum of a number and its opposite is always zero.

Absolute value: The value of a positive or negative number without its sign is called its absolute value. For example, the absolute value of both $+10$ and -10 is 10.

Positive expression and negative expression:

You know that '+' sign means positive sign and '-' sign means negative sign. That is why expressions with '+' sign are called positive expressions and those with '-' sign are called negative expressions. Generally, '+' sign is not attached with positive expression. If there is no sign in front of an expression it is taken as a positive expression. For example,

5 metres, Rs y , a hours, p kilograms are all positive expressions, where as $-Rs\ 10$, $-2\ kg$, $-7\ metres$, $-x\ hours$ are all negative expressions.

Example 1. What do the following expressions mean:

- (i) a profit of $-Rs\ 15$. (ii) $-5\ km$. towards east.
(iii) ascending $-30\ metres$.

Ans: (i) a loss of Rs 15.
(ii) 5 km towards west
(iii) descending 30 metres.

Example 2. (i) If a profit of Rs 10 is written as -10 , how will you write a loss of Rs 15?

(ii) If a distance of 25 km towards west is written as 25 km, how will you write a distance of 10 km towards east?

Ans: (i) 15 (ii) $-10\ km$.

Example 3. Write the opposite of the following expressions:

- (i) an expenditure of Rs 10 (ii) a loss of Rs 110
(iii) a profit of Rs 15 (iv) descending 25 m.
(v) a profit of $-Rs\ 5$.

Ans: (i) an income of Rs 10
(ii) a profit of Rs 110
(iii) a loss of Rs 15
(iv) ascending 25 m.
(v) a profit of Rs 5.

Example 4. Write the absolute values of the following numbers:

- (i) $+3$ (ii) -11 (iii) -10 (iv) -20 (v) -15 .

Ans: (i) 3 (ii) 11 (iii) 10 (iv) 20 (v) 15.

Exercise 3

1. What do the following expressions mean?
(i) a loss of -Rs. 10 (ii) increase of weight by -20 kg.
(iii) fall of temperature by -5° (iv) -3m below (v) -12 km. to the east (vi) Kamal earns -Rs 5 more than Karim.
2. Write the opposites of the following expressions:
(i) an income of Rs 5 (ii) a loan of Rs 12 (iii) 3 km. to the north (iv) decreasing weight by 2 kg. (v) -5 km. to the east (vi) a fall of temperature by -3° .
3. Show the position of the following points on the number-axis:
(i) +5, (ii) -3, (iii) -5, (iv) +6, (v) -10, (vi) -4.
4. Arrange the following numbers in the ascending order of their values:
(i) -2, 5, -10.
(ii) -12, -13, -16, 0.
(iii) 5, 7, -8, -10.
5. Write the absolute values of the following numbers:
(i) -5, (ii) +10, (iii) -100, (iv) +15, (v) -200.
6. Starting from 1st January what will be the date after -10 days?
7. A shopkeeper buys an article at Rs 60 and sells it at a profit of -Rs 5. What is the selling price?

2.2 First four rules:

You have learnt in arithmetic how to add, subtract, multiply and divide. In arithmetic the numbers are all positive, but in algebra the numbers may be both positive and negative. We will now discuss how these four rules can be applied in case of directed numbers.

Addition of whole numbers**(i) Finding sum of two positive numbers:**

You can easily understand the method of addition with the help of number-axis.

Suppose, you will have to find the sum of $(+4)$ and $(+2)$.

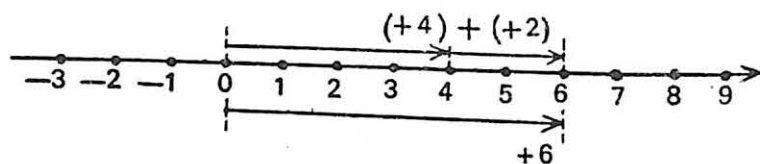


Fig. 6

Moving four units to the right of O we get the point $+4$. After this we have two more units in the same direction (Fig. 6). Now if you count, you will find that we have moved 6 units from the point O in the positive direction; that is, that point is $+6$. Thus we get

$$(+4) + (+2) = (+6)$$

Similarly see that (Fig. 7)

$$(+3) + (+4) = (+7)$$

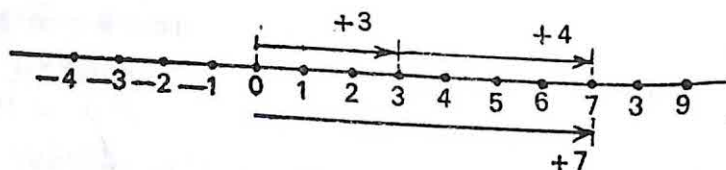


Fig. 7

Further examples are

$$(+2) + (+3) = (+5)$$

$$(+6) + (+3) = (+9)$$

$$(+5) + (+5) = (+10)$$

$$(+7) + (+4) = (+11)$$

Thus you find that if you add the absolute values of two positive numbers and put a '+' sign before the result, you will get the sum of those two numbers.

(ii) Finding sum of a positive number and a negative number:

Suppose that we are to find out the sum of $(+5) + (-2)$. In this case also the result may be easily found out with the help of number-axis. If you move 5 units to the right from the point O you will get the point +5. The meaning of adding (-2) with it is the same as moving 2 units in the opposite direction, that is, to the left starting from the above point (Fig. 8). Now which side of the point O is the point you reach and how many units away from it?

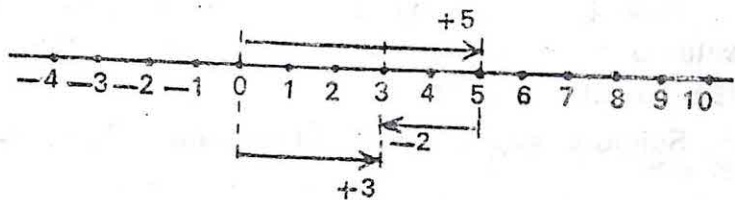


Fig. 8

See that you have reached a point 3 units from O in the positive direction.

Thus we find that

$$(+5) + (-2) = (+3).$$

In the same manner see that (Fig. 9)

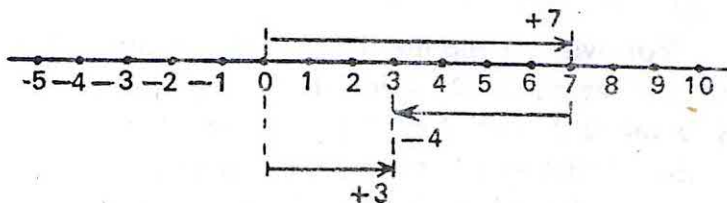


Fig. 9

$$(+7) + (-4) = (+3).$$

Further examples are

$$(+5) + (-1) = (+4)$$

$$(+10) + (-6) = (+4)$$

$$(+7) + (-5) = (+2)$$

$$(+12) + (-11) = (+1)$$

Thus you find that between a positive and a negative numbers if the absolute value of the positive number is greater than that of the negative number, the absolute value of their **sum** will be the **difference** of the absolute values of the two numbers and the sum will be positive i.e., it will have '+' sign.

Now, let us see what happens when the absolute value of the negative number is greater than that of the positive number.

Suppose, we are to find out the sum $(-7) + (+2)$.

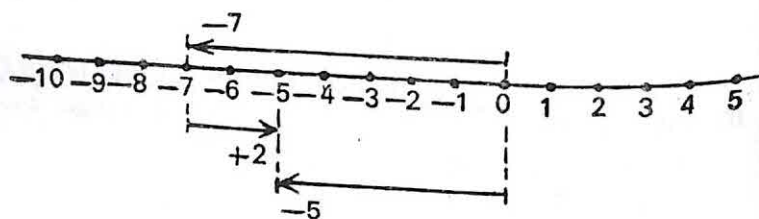


Fig. 10

You will get the number -7 if you move 7 units in the negative direction from the point O. If you move 2 units to the right from that point you will find that you have reached a point 5 units from the point O and to the left of it (Fig. 10). Thus you find that

$$(-7) + (+2) = (-5)$$

In the same manner see that (Fig. 11)

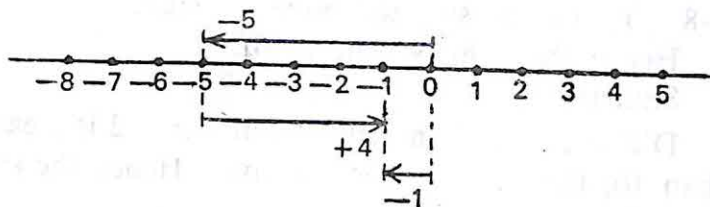


Fig. 11

$$(-5) + (+4) = (-1).$$

Further examples are

$$(-8) + (+2) = (-6)$$

$$(-10) + (+6) = (-4)$$

$$(-12) + (+5) = (-7)$$

$$(-9) + (+4) = (-5)$$

From these examples, you find that between a positive and a negative numbers if the absolute value of the negative number is greater than that of the positive number, the absolute value of their sum will be equal to the **difference** of the two absolute values and the sum will be negative i.e., it will have '-' sign.

From the above two rules we may say that if a positive number and a negative number are added together the absolute value of the sum will be equal to the different of the absolute values of the two numbers and the sum will have the sign of the number whose absolute value is greater. For example,

$$(-8) + (+2) = (-6)$$

Difference of 8 and 2 is 6; 8 is greater than 2

and so the sum will have the same sign as that of -8 . That is the sum will have ' $-$ ' sign.

Hence the sum will be (-6) .

Similarly, $(-10) + (+12) = (+2)$.

Difference of 12 and 10 is 2, but since 12 is greater than 10, the sum will be positive. Hence the sum is $(+2)$.

In general, we may say that

$$\begin{aligned} (+a) + (-b) &= a - b \text{ if } a > b, \\ &= -(b - a) \text{ if } b > a. \end{aligned}$$

You have already seen that the sum of a number and its opposite is zero, as for example—

$$(+3) + (-3) = 0$$

$$(-9) + (+9) = 0$$

In general, we may write—

$$a + (-a) = 0,$$

$$(-x) + x = 0, \text{ etc.}$$

Note: $0 + (+3) = (+3)$

$$(-3) + 0 = (-3)$$

$$0 + 0 = 0$$

That is if '0' is added to a positive or negative number the number remains unchanged.

(iii) Finding sum of two negative numbers:

Suppose that we will have to find out the sum of (-2) and (-3) . On the number-axis if you go two units to the left of 0 you get (-2) .

If you go 3 more units from that point in the same direction, then see that you have reached the point -5 (Fig. 12).

That is $(-2) + (-3) = (-5)$.

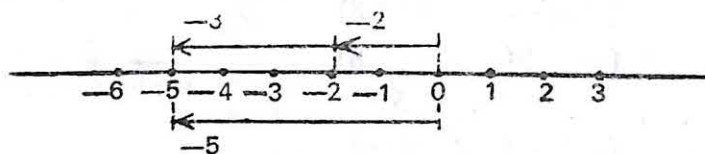


Fig. 12

In the same manner, see (Fig. 13)

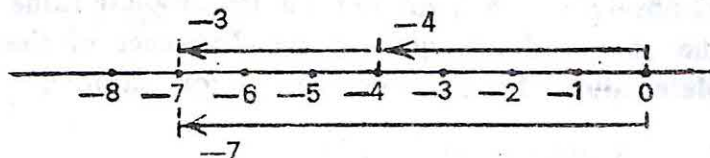


Fig. 13

$$(-4) + (-3) = (-7)$$

Further examples are:

$$(-2) + (-1) = (-3)$$

$$(-5) + (-3) = (-8)$$

$$(-4) + (-1) = (-5)$$

$$(-4) + (-7) = (-11)$$

You can see from the above examples that if two negative numbers are added together the sum will be negative and the absolute value of the sum will be equal to the sum of the absolute values of two numbers.

Rule for addition of directed numbers :

1. The sum of two numbers of same sign (both positive or both negative) will have the sign of the given numbers and the absolute value of the sum will be equal to the sum of the absolute values of the given

numbers. For example,

$$(+5) + (+3) = (+8)$$

$$(-5) + (-3) = (-8)$$

$$2 + 3 = 5$$

$$(-3) + (-2) = (-5) \text{ etc.}$$

2. The sign of the sum of a positive and a negative number will be the same as that of the number whose absolute value is greater and the absolute value of the sum will be equal to the difference of the absolute values of the two numbers. For example,

$$(+6) + (-3) = (+3)$$

$$(+4) + (-10) = (-6)$$

$$(+5) + (-1) = (+4)$$

$$(+1) + (-5) = (-4) \text{ etc.}$$

Algebraic sum: Sum of several numbers is found by putting the numbers one after another with their signs. This sum is called algebraic sum of the numbers.

Example 1. Add on the number-axis:

(i) $(+7) + (2)$

(ii) $(+3) + (-6)$

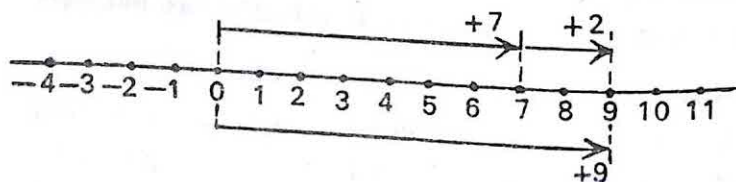


Fig. 14

$$(+7) + (+2) = (+9)$$

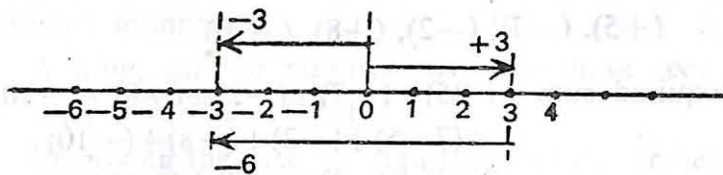


Fig. 15

$$(+3) + (-6) = (-3)$$

Example 2. Find the sum:

(i) $5, -3$

(ii) $8, -10$

(iii) $12, -5$

(iv) $-3, -2.$

Ans:

(i) $5 + (-3) = (5 - 3) = 2$

(ii) $8 + (-10) = -(10 - 8) = -2$

(iii) $12 + (-5) = (12 - 5) = 7$

(iv) $(-3) + (-2) = -(3 + 2) = -5.$

Example 3. Find the sum:

(i) $(-5), (-2), +6$

(ii) $(-1), (-3), +8, +2$

Ans:

(i) Required sum

$$= (-5) + (-2) + 6$$

$$= -(5 + 2) + 6$$

$$= (-7) + 6 = -(7 - 6)$$

$$= -1$$

(ii) Required sum

$$= (-1) + (-3) + (+8) + (+2)$$

$$= -(1 + 3) + (8 + 2)$$

$$= (-4) + 10$$

$$= +(10 - 4) = +6$$

Example 4. Find the algebraic sum of

$(+5), (-7), (-2), (+8), (-10)$.

$$\begin{aligned}
 \text{Required sum} &= (+5) + (-7) + (-2) + (+8) + (-10) \\
 &= -(7-5) + (-2) + (+8) + (-10) \\
 &= (-2) + (-2) + (+8) + (-10) \\
 &= -(2+2) + (+8) + (-10) \\
 &= (-4) + (+8) + (-10) \\
 &= (8-4) + (-10) \\
 &= (+4) + (-10) = -(10-4) = -6
 \end{aligned}$$

Exercise 4

1. Add on the number-axis:

- (i) $(+2), (+4)$ (ii) $(+5), (-1)$ (iii) $(+2), (-5)$
 (iv) $(-4), (-2)$.

2. Add:

- (i) $(-2), 5$ (ii) $(-5), (-3)$ (iii) $8, (-10)$
 (iv) $(-3), (-6)$ (v) $(-5), 12$.

3. Simplify:

- (i) $2 + (-3) + 5$ (ii) $3 + (-5) + (-6)$
 (iii) $1 + (-2) + (-3) + 4$ (iv) $6 + (-6) + 7 + (-8)$.

4. Find the algebraic sum:

- (i) $(-1), (-2), (+3)$ (ii) $(+5), (-2), (-3)$
 (iii) $(-8), (-2), (-1), (+5)$ (iv) $(-12), (-4), (-1)$.

Substraction

You know that if 5 is added to 7 we get 12 and so if 5 is subtracted from 12 the result will be 7.

Now let us see how we can use this idea in case of directed numbers.

Adding, on the number-axis, you have seen

$$(+7) + (+5) = (+12)$$

Removing the first '+' sign for positive number, we may write

$$7 + 5 = 12$$

Hence, from the idea of subtraction, we get,

$$12 - 5 = 7$$

Again, you have also seen that

$$12 + (-5) = 7$$

Hence, we find that

$$12 - 5 = 12 + (-5)$$

and $-5 = +(-5).$

But -5 is the opposite of 5, thus **subtracting 5 from 12 means adding opposite of 5 with 12.**

Let us take another example

$$10 + 2 = 12$$

Hence, from the idea of subtraction,

$$12 - 2 = 10$$

Again, adding on the number-axis, we get

$$12 + (-2) = 10$$

$$\therefore 12 - 2 = 12 + (-2)$$

$$\therefore -2 = +(-2)$$

In general, we may write

$$x - y = x + (-y),$$

that is, subtracting a positive expression from any expression means adding to that expression

a negative expression of equal absolute value, or adding to that expression its opposite expression.

For example,

$$10 - 8 = 10 + (-8) = 2$$

$$25 - 20 = 25 + (-20) = +5$$

$$(-4) - 5 = (-4) + (-5) = -9$$

$$(-12) - 1 = (-12) + (-1) = -13 \text{ etc.}$$

Again see,

$$6 + (-4) = 2$$

Thus according to the idea of subtraction

$$2 - (-4) = 6.$$

Again, it can be easily shown on the number-axis, that

$$2 + 4 = 6$$

$$\therefore 2 - (-4) = 2 + 4$$

$$\text{and } -(-4) = +4.$$

Just like this, we may show that

$$9 - (-1) = 9 + 1,$$

$$\text{and } -(-1) = +1.$$

In general, we may write that

$$x - (-y) = x + y$$

Note that $-(-y) = y$.

Thus, subtracting a negative expression from any expression means adding to that expression a positive expression whose absolute value is equal to that of the negative expression; that is adding to that expression its opposite expression.

Further examples are

$$(-3) - (-1) = (-3) + 1 = -2$$

$$(-10) - (-5) = (-10) + 5 = -5$$

$$2 - (-5) = 2 + 5 = 7$$

$$3 - (-1) = 3 + 1 = 4.$$

From the above discussion we get the general rule of subtraction as follows :

Subtracting one expression from another means adding to the first expression the opposite of the second expression.

Note: (1) $a - 0 = a$

(2) $0 - a = -a$

(3) $0 - 0 = 0.$

Example: Subtract:

(i) $(+2)$ from $(+5)$

(ii) (-3) from $(+10)$

(iii) (-7) from (-5)

Ans : (i) $(+5) - (+2) = 5 + (-2) = 5 - 2 = 3$

(ii) $(+10) - (-3) = 10 + 3 = 13$

(iii) $(-5) - (-7) = (-5) + 7 = 7 - 5 = 2.$

Exercise 5

1. Subtract second from the first:

(i) 15, 6

(ii) -10, 8

(iii) -2, 3

(iv) 4, -5

(v) -50, -10

(vi) +15, -10

(vii) -12, +3

(viii) -10, +10

(ix) +5, +5

(x) -6, +10.

2. How much is to be added to the first to get the second?

(i) -10, -15

(ii) 4, -12

(iii) 16, 12

(iv) 25, -32

(v) 100, 0.

Multiplication

You know,

$$5 \times 3 = 15$$

again $5 + 5 + 5 = 15$.

That is, adding 5 three times is same as multiplying 5 by 3. This means that the operation of addition is done in a shorter way by multiplication. In arithmetic you have learnt multiplication of positive numbers. But if we consider both positive and negative numbers, there may be three types of multiplication.

(1) Multiplying positive number by positive number—

for example, 5×3 , 4×5 , etc.;

(2) Multiplying positive number by negative number or negative number by positive number—

for example, $2 \times (-4)$, $(-4) \times 5$, etc.;

and (3) Multiplying negative number by negative number—

for example, $(-3) \times (-2)$, $(-3) \times (-5)$, etc.

(1) Multiplying positive number by positive number :

Suppose, we want to find out the value of $(+3) \times (+2)$.

We can write

$$\begin{aligned} (+3) \times (+2) &= 3 \times (+2) \\ &= (+2) + (+2) + (+2) \\ &= +6 \end{aligned}$$

[We get this on the number-axis by adding 2 units 3 times in the positive direction (Fig. 16)].
That is $3 \times 2 = 6$.

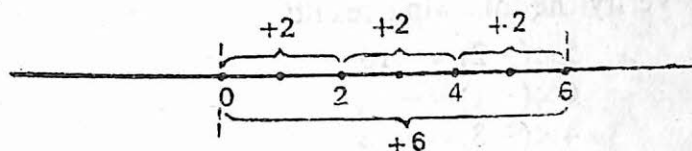


Fig. 16

Thus we find that the product of two positive numbers will be like the product of whole numbers in arithmetic.

Further examples are

$$(+3) \times (+4) = +12$$

$$(+5) \times (+3) = +15$$

$$(+10) \times (+2) = +20 \text{ etc.}$$

(2) Multiplying positive number by negative number or negative number by positive number :

Suppose, we want to find out the value of $(+4) \times (-2)$.

We can write

$$\begin{aligned} (+4) \times (-2) &= 4 \times (-2) \\ &= (-2) + (-2) + (-2) + (-2) \\ &= (-8). \end{aligned}$$

That is $4 \times (-2) = -8$

See on the number-axis (Fig. 17),

$$[4 \times (-2) = -8]$$

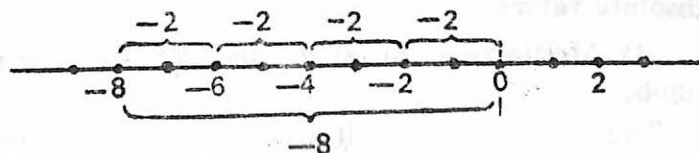


Fig. 17

Verify the following results on the number-axis.

$$5 \times (-2) = -10$$

$$6 \times (-1) = -6$$

$$4 \times (-3) = -12$$

Again see,

$$\begin{aligned} (-4) \times (+2) &= (-4) \times 2 \\ &= (-4) + (-4) \\ &= (-8). \end{aligned}$$

That is $(-4) \times 2 = -8$.

See on the number-axis (Fig. 18),

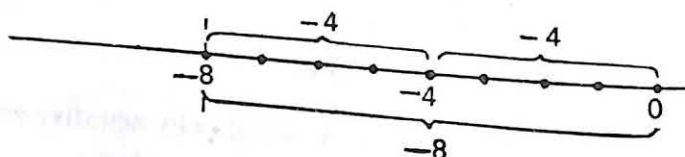


Fig. 18

$$[4 \times (-2) = -8]$$

Further examples are,

$$(-3) \times 3 = -9$$

$$(-5) \times 2 = -10$$

$$(-6) \times 1 = -6$$

Verify these results by drawing a number-axis.

Thus we find that the product of a positive number with a negative number or a negative number with a positive number will be negative and the absolute value of the product is the product of the two absolute values.

(3) Multiplying negative number by negative number :

The nature of multiplying negative number by negative number is worth noticing. That is

why we try to understand this process by two real examples.

Suppose that there is a stock of some clothes in the shop of a co-operative society. From that 100 pairs of clothes are sold everyday. You can easily understand that 5 days before a certain day there were 500 pairs more clothes in the shop than that on the particular day.

Now, from the concept of opposite number if increase in stock is taken as positive, decrease in stock will have to be taken as negative. That is, the decrease of stock by 100 pairs of clothes per day can be taken as increase of stock by -100 pairs per day. Similarly, the time '5 days before' may be written as ' -5 days after'.

Thus 5 days before, excess clothes that were in the stock of the shop

$$=(-5) \times (-100) \text{ pairs.}$$

$$\text{That is } (-5) \times (-100) = 500.$$

See another example.

Suppose that you have a box for saving coins. You spend 25 paise from that every day. 4 days before now there was 4×25 paise or Re 1 more than what is there in the box now.

Again, spending 25 paise everyday can be taken as saving -25 paise. Then 4 days before, that is -4 days after, the saving in the box will be $(-4) \times (-25)$ paise.

$$\text{That is } (-4) \times (-25) = 100.$$

From the above two examples you find that the product of two negative numbers is a positive

number. You can understand this with the help of number-axis also.

You know, $4 \times (-2) = -8$

Therefore, opposite number of $4 \times (-2) =$
opposite number of -8 .

That is $-4 \times (-2) = 8$.

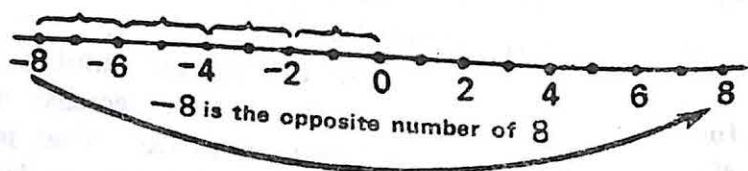


Fig. 19

Thus we find that the product of two negative numbers is positive and its absolute value is equal to the product of the absolute values of the two numbers.

Further examples are:

$$(-2) \times (-3) = 6$$

$$(-1) \times (-1) = 1$$

$$(-8) \times (-1) = 8$$

$$(-5) \times (-10) = 50 \text{ etc.}$$

Note : If any number is multiplied by zero the product will be zero.

For example, $a \times 0 = 0$

$$-b \times 0 = 0.$$

$$0 \times 0 = 0.$$

Example 1. Multiply:

(i) 5 by (-3)

(ii) (-4) by 5

(iii) (-2) by (-4)

(iv) (-2) by (-6)

- Ans:**
- (i) $5 \times (-3) = -15$
 - (ii) $(-4) \times 5 = -20$
 - (iii) $(-2) \times (-4) = 8$
 - (iv) $(-2) \times (-6) = 12.$

Exercise 6

Multiply the first number by the second number:

1. 4, (-3)
 2. (-3), 2
 3. 2, (-5)
 4. 8, 12
 5. (-5), (-4)
 6. (-3), (-5)
 7. (-10), 3
- Multiply:
8. $3 \times (-2) \times (-4)$
 9. $(-5) \times (-2) \times 10$
 10. $3 \times 5 \times (-2) \times (-1)$
-

Index rule of multiplication :

You have learnt in class VI how to express a number in terms of its factors.

For example, $12 = 2 \times 2 \times 3$
 $= 2^2 \times 3$

$16 = 2 \times 2 \times 2 \times 2$
 $= 2^4.$

In 12 the factor 2 occurs twice and the factor 3 occurs once. That is why we write

$$12 = 2^2 \times 3.$$

See that a small 2 is written on the right top of 2. This means that the factor 2 occurs twice in 12. Here 2^2 means 2×2 . If any expression is multiplied by itself a number of times, then instead of writing the expression side by side in the product, the number (of times it is multiplied) is written on

the right hand side above the expression to denote the number of factors. The number that is written above is called the index or power of expression.

For example, in 2^2 the index of 2 is 2,

in 2^4 the index of 2 is 4,

in 3^3 the index of 3 is 3, etc.

In case of symbols representing numbers, this rule will also be applicable in the same manner.

For example,

$$a \times a = a^2$$

$$a \times a \times a = a^3$$

$$x \times x \times x \times x = x^4 \text{ etc.}$$

In a^2 the index of a is 2,

in a^3 the index of a is 3,

in x^4 the index of x is 4, etc.

Note:

$$a^1 = a,$$

$$x^1 = x, \text{ etc.}$$

Generally the index 1 is not written.

Now see,

$$a^2 \times a^3 = (a \times a) \times (a \times a \times a)$$

$$= a \times a \times a \times a \times a$$

$$= a^5 = a^{2+3}$$

Therefore, $a^2 \times a^3 = a^{2+3}$

Similarly, $x^3 \times x^4 = (x \times x \times x) \times (x \times x \times x \times x)$

$$= x \times x \times x \times x \times x \times x \times x = x^7.$$

Therefore, $x^3 \times x^4 = x^7 = x^{3+4}.$

In general,

$$a^m = (a \times a \times a \times \dots \text{upto } m \text{ terms})$$

$$a^n = (a \times a \times a \times \dots \text{upto } n \text{ terms})$$

$$\begin{aligned} \therefore a^m \times a^n &= (a \times a \times a \times \dots \text{upto } m \text{ terms}) \\ &\quad \times (a \times a \times a \times \dots \text{upto } n \text{ terms}) \\ &= a \times a \times a \times \dots \text{upto } (m+n) \text{ terms} \\ &= a^{m+n}. \end{aligned}$$

$$\therefore a^m \times a^n = a^{m+n}.$$

Example 1. Find the value of the following expressions:

$$(i) \ 3^3 \quad (ii) \ (-2)^2 \quad (iii) \ (-1)^3.$$

$$\text{Ans:} \quad (i) \ 3^3 = 3 \times 3 \times 3 = 27$$

$$(ii) \ (-2)^2 = (-2) \times (-2) = 4$$

$$\begin{aligned} (iii) \ (-1)^3 &= (-1) \times (-1) \times (-1) \\ &= 1 \times (-1) = -1. \end{aligned}$$

Example 2. Multiply:

$$(i) \ x \times x^2$$

$$(ii) \ a^2 \times a^3$$

$$(iii) \ -a^3 \times a^4$$

$$(iv) \ (-x^2) \times (-x^5)$$

$$\text{Ans:} \quad (i) \ x \times x^2 = x^1 \times x^2 = x^{1+2} = x^3$$

$$(ii) \ a^2 \times a^3 = a^{2+3} = a^5$$

$$(iii) \ -a^3 \times a^4 = -(a^3 \times a^4) = -a^{3+4} = -a^7$$

$$(iv) \ (-x^2) \times (-x^5) = +(x^2 \times x^5) = x^{2+5} = x^7.$$

Example 3. If $a = -1$, $b = 2$, $c = -2$, then find the values of the following expressions:

$$(i) \ ab^2$$

$$(ii) \ a^2b$$

$$(iii) \ ab^2c$$

$$(iv) \ a^2 + b^2 + c^2.$$

Ans :

$$(i) \quad ab^2 = (-1) \times 2^2 = -1 \times (2 \times 2) = (-1) \times 4 = -4$$

$$(ii) \quad a^2b = (-1)^2 \times 2 = (-1) \times (-1) \times 2 = 1 \times 2 = 2$$

$$(iii) \quad ab^2c = (-1) \times 2^2 \times (-2) = (-1) \times 2 \times 2 \times (-2) \\ = (-2) \times (-4) = 8$$

$$(iv) \quad a^2 + b^2 + c^2 = (-1)^2 + 2^2 + (-2)^2 \\ = (-1) \times (-1) + 2 \times 2 + (-2) \times (-2) \\ = 1 + 4 + 4 \\ = 9.$$

Exercise 7

Multiply:

$$1. \quad x^2 \times x^3 \quad 2. \quad x \times x^5 \quad 3. \quad a^7 \times a^8 \quad 4. \quad a^{10} \times a^3$$

$$5. \quad (-a^2) \times (-a) \quad 6. \quad a^3 \times (-a) \quad 7. \quad 2^4 \times 2^2 \quad 8. \quad 3^3 \times 3^2$$

$$9. \quad 5^2 \times 5^3 \quad 10. \quad (-5)^2 \times (-4)^3.$$

$$11. \quad \text{If } a=1, b=2, c=-3, \text{ find the values of (i) } a^2+b^2+c^2, \\ (ii) \quad a^3+b^3+c^3.$$

$$12. \quad \text{If } x=-1, y=-2, \text{ find the values of (i) } x^2y^2, (ii) \quad x^4y^4, \\ (iii) \quad x^2+y^4.$$

$$13. \quad \text{If } a=2, b=-1, c=1, \text{ find the values of (i) } abc^2, \\ (ii) \quad a^2b^2c^2, \text{ and (iii) } a^2b^3c^3.$$

Division

You know,

$$12 \times 3 = 36.$$

$$\text{Therefore } 36 \div 3 = 12$$

$$\text{and } 36 \div 12 = 3.$$

Thus if any number is the product of two other numbers, then dividing the first number by any one of the two numbers we get the other number as the quotient.

In general, we may say that if a and b are two numbers and if

$$a \times b = c$$

$$\text{then } c \div b = a \text{ and } c \div a = b.$$

$$\text{But } c = a \times b = ab$$

$$\text{Therefore, } c \div b = ab \div b = a$$

$$\text{and } c \div a = ab \div a = b.$$

Writing in the form of fraction, we will get

$$\frac{ab}{b} = a \text{ and } \frac{ab}{a} = b.$$

At the time of multiplication you have seen that

$$7 \times (-3) = -21.$$

From the above discussion we may write

$$-21 \div 7 = -3$$

$$\text{or, } \frac{-21}{7} = -3.$$

-21 is a negative number and 7 is a positive number; thus if a negative number is divided by a positive number the quotient will be negative and the absolute value of the quotient is the same as the quotient of the absolute values of the two numbers.

Again, from the same multiplication $7 \times (-3) = -21$, we find that $(-21) \div (-3) = 7$

$$\text{or } \frac{-21}{-3} = 7.$$

This means that if a negative number is divided by another negative number, the quotient will be positive and the absolute value of the quotient will be equal to the quotient of the absolute values of the two numbers.

From the rule of multiplication you have also seen that

$$(-3) \times (-7) = 21.$$

Thus from the concept of division,

$$21 \div (-3) = -7$$

and $21 \div (-7) = -3,$

or, $\frac{21}{-3} = -7$ and $\frac{21}{-7} = -3.$

That is if a positive number is divided by a negative number, the quotient will be negative and the absolute value of the quotient will be equal to the quotient of the absolute values of the two numbers.

Further examples are:

(i) $\frac{25}{-5} = -5$

(ii) $\frac{-30}{10} = -3$

(iii) $\frac{-50}{-25} = 2$

(iv) $\frac{-15}{3} = -5$ etc.

Example 1. Divide the first number by the second:

(i) $-15, 3$ (ii) $25, -5$ (iii) $16, -8$
 (iv) $-16, -2.$

Ans: (i) $(-15) \div 3 = \frac{-15}{3} = -5$

$$(ii) \quad 25 \div (-5) = \frac{25}{-5} = -5$$

$$(iii) \quad 16 \div (-8) = \frac{16}{-8} = -2$$

$$(iv) \quad (-16) \div (-2) = \frac{-16}{-2} = 8$$

Rule for signs in case of division:

(1) If dividend and divisor are both of the same sign (that is if both are positive or both are negative), the sign of the quotient will be positive.

(2) If dividend and divisor are of opposite signs (that is, if one is positive and the other is negative), the sign of the quotient will be negative.

Rule for division in case of zero (0):

(1) If zero is divided by any number the quotient will be zero. For example

$$(i) \quad 0 \div 3 = 0$$

$$(ii) \quad 0 \div a = 0 \text{ etc.}$$

But (2) division of any number by zero does not have any sense. That is, the value of divisor can never be zero.

Exercise 8

Divide first number by second:

- | | | | |
|---------------|-------------|--------------|-------------|
| 1. 16, -4 | 2. -14, 2 | 3. -15, 3 | 4. 100, -10 |
| 5. -120, 15 | 6. 144, -16 | 7. -125, -25 | |
| 8. -121, -11. | | | |
-

Chapter 3

Laws

Use of commutative, associative and distributive laws and of brackets.

3.1 Commutative law :

You know, $5+3=8$

again, $3+5=8$

Then we can write

$$5+3=3+5$$

That is, we get the same sum by adding 3 to 5 or 5 to 3. Similarly,

$$5+4=4+5$$

$$6+4=4+6 \text{ etc.}$$

Generalising, we can write

$$a+b=b+a.$$

Even if we interchange the positions of a and b the sum remains unchanged. This law is called **Commutative law** of addition. Not only in case of addition, but in case of multiplication also this commutative law will be applicable. For example,

(i) $4 \times 5 = 20$

and $5 \times 4 = 20$

Therefore, $4 \times 5 = 5 \times 4$.

(ii) $2 \times (-3) = -6$

and $(-3) \times 2 = -6$

Therefore, $2 \times (-3) = (-3) \times 2$.

$$(iii) \quad (-4) \times (-3) = 12$$

$$\text{and } (-3) \times (-4) = 12$$

Therefore, $(-4) \times (-3) = (-3) \times (-4)$.

In general, we can write, $a \times b = b \times a$.

That is ab and ba are equal. Examine and see whether **commutative law** is applicable in case of subtraction and division.

3.2 Associative law:

You know how to find the sum of three numbers. After adding first two numbers, the third number is added to that sum, the required sum is obtained. That is, first number + second number + third number = (first number + second number) + third number. Writing first number + second number within bracket, we mean that first we have found the sum of the first and second numbers.

$$\begin{aligned}\text{For example, } 5 + 7 + 9 &= (5 + 7) + 9 \\ &= 12 + 9 \\ &= 21\end{aligned}$$

Now, instead of taking the sum of first two numbers first and then adding the third number, if we add the second and the third numbers first and then add this sum to the first number, we will get the same result. For example:

$$\begin{aligned}5 + 7 + 9 &= 5 + (7 + 9) \\ &= 5 + 16 \\ &= 21.\end{aligned}$$

Thus, we get

$$(5 + 7) + 9 = 5 + (7 + 9).$$

Further examples are

$$1+(2+3)=(1+2)+3$$

$$5+(6+9)=(5+6)+9$$

$$5+\{(-3)+2\}=\{5+(-3)\}+2.$$

Applicable form of this rule is as follows:

If an expression denotes successive addition, then the value of the sum does not change when some successive terms are put into brackets in any manner whatsoever.

In general, we get

$$a+(b+c)=(a+b)+c.$$

This law is called **associative law of addition**. For more than three numbers also this law will be applicable. Test the applicability of the law taking four numbers.

Like addition this law also holds good in case of multiplication. For example

$$2 \times 5 \times 6 = (2 \times 5) \times 6 = 10 \times 6 = 60$$

$$\text{Again, } 2 \times (5 \times 6) = 2 \times 30 = 60$$

Thus, we get

$$(2 \times 5) \times 6 = 2 \times (5 \times 6)$$

Further examples are

$$2 \times \{(-5) \times 2\} = \{2 \times (-5)\} \times 2$$

$$5 \times \{(-4) \times (-3)\} = \{5 \times (-4)\} \times (-3).$$

In general, we may write

$$a \times (b \times c) = (a \times b) \times c.$$

This law is called **associative law of multiplication**. Test and see if associative law is applicable in case of subtraction and division.

3.3 Distributive law:

Suppose that a person will buy 3 exercise books and 3 pens.

Let the price of each exercise book be Re 1 and that of each pen be Rs 3. Then what will be the price of 3 exercise books and 3 pens? We can proceed in two ways,

$$\text{Price of 1 exercise book} = \text{Re } 1$$

$$\text{Price of 1 pen} = \text{Rs } 3$$

$$\begin{aligned} \text{Price of 1 exercise book} + \text{Price of 1 pen} \\ = \text{Re } 1 + \text{Rs } 3. \end{aligned}$$

$$\begin{aligned} \therefore \text{Price of 3 exercise books and 3 pens} \\ = 3 \times (\text{price of 1 exercise book} + \text{price of 1 pen}) \\ = 3(\text{Re } 1 + \text{Rs } 3) = \text{Rs } 12. \end{aligned}$$

$$\begin{aligned} \text{Again, price of 3 exercise books} &= \text{Rs } (3 \times 1) \\ \text{and price of 3 pens} &= \text{Rs } (3 \times 3) \end{aligned}$$

$$\begin{aligned} \therefore \text{price of 3 exercise books and 3 pens} \\ = \text{Rs } (3 \times 1) + \text{Rs } (3 \times 3) = \text{Rs } 12. \end{aligned}$$

Thus we find that

$$3(\text{Re } 1 + \text{Rs } 3) = \text{Rs } (3 \times 1) + \text{Rs } (3 \times 3) = \text{Rs } 12.$$

$$\text{Writing without unit, } 3(1+3) = 3 \times 1 + 3 \times 3 = 12.$$

Again, take three numbers, for example 2, 5, 7.

$$\text{Then } 2(5+7) = 2 \times 12 = 24$$

$$\text{Again } 2 \times 5 = 10$$

$$2 \times 7 = 14$$

$$\text{Therefore, } 2 \times 5 + 2 \times 7 = 10 + 14 = 24.$$

$$\text{Then, } 2(5+7) = 2 \times 5 + 2 \times 7.$$

Further examples are

$$4(2+1) = 4 \times 2 + 4 \times 1$$

$$2(3+2) = 2 \times 3 + 2 \times 2$$

This law is applicable in case of negative numbers also.

For example, let -3 , 2 , 4 be three numbers.

$$\begin{aligned}\text{Now } & (-3)(2+4) \\ & = (-3) \times 6 = -18\end{aligned}$$

$$\begin{aligned}\text{Again, } & (-3) \times 2 = -6 \\ & (-3) \times 4 = -12\end{aligned}$$

$$\therefore (-3) \times 2 + (-3) \times 4 = -6 - 12 = -18.$$

Thus we find,

$$(-3)(2+4) = (-3) \times 2 + (-3) \times 4.$$

In general, we may write

$$a(b+c) = ab+ac.$$

In words, we may state this law as follows:

$$\begin{aligned}\text{first number} \times (\text{second number} + \text{third number}) \\ = \text{first number} \times \text{second number} + \text{first number} \\ \times \text{third number}.\end{aligned}$$

This law is the distributive law of multiplication.

Note that both the numbers inside the bracket have been multiplied separately by the number outside the bracket and then the products have been added, that is the multiplier outside the bracket has been distributed in each number inside the bracket.

Again, note that if the '+' sign inside the bracket is replaced by '-' sign, then also this law will be applicable. For example

$$15(6-2) = 15 \times 4 = 60$$

$$\text{Now } 15 \times 6 = 90$$

$$15 \times 2 = 30$$

Therefore $15 \times 6 - 15 \times 2 = 90 - 30 = 60$

Hence $15(6-2) = 15 \times 6 - 15 \times 2$

Hence, in general, we may write

$$a(b-c) = ab - ac.$$

Note: (1) If there are more than two numbers inside the bracket the law will be equally applicable.

For example, $a(b+c+d) = ab+ac+ad$

$$a(b+c-d) = ab+ac-ad$$

(2) In $a(b+c)$; a is to the left of $(b+c)$ as a multiplier. The result will be same, if a is to the right of the brackets as a multiplier.

$$(b+c)a = ba+ca.$$

Distributive law of division:

$$(20+10) \div 5 = 30 \div 5 = 6$$

Again, $20 \div 5 = 4$ and $10 \div 5 = 2$

$$\therefore 20 \div 5 + 10 \div 5 = 4 + 2 = 6$$

Hence $(20+10) \div 5 = 20 \div 5 + 10 \div 5$

In general, we may write

$$(a+b) \div c = a \div c + b \div c$$

Writing in the form of fraction, we get

$$\frac{20+10}{5} = \frac{20}{5} + \frac{10}{5}$$

Further examples are

$$\frac{50+40}{10} = \frac{50}{10} + \frac{40}{10}$$

$$\frac{15+45+75}{15} = \frac{15}{15} + \frac{45}{15} + \frac{75}{15}$$

In general, we may write

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}.$$

Note that the distributive law of division holds only when the divisor is on the right hand side; that is

$$a \div (b+c) \neq a \div b + a \div c$$

$$\text{or, } \frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$$

Take some numbers and test this inequality.

3.4 Use of brackets:

In algebra brackets are used in the same manner as in arithmetic. The whole portion inside a bracket is taken as one expression. For example, if we write $3(2+3)$ we mean that the sum of 2 and 3 has been multiplied by 3. Similarly, $x(y+z)$ means the product of x and the sum of y and z .

‘+’ and ‘-’ signs before brackets have some specialities. Note the following examples carefully.

Example 1. Add the sum of 2 and 5 with 3.

$$\begin{aligned} \text{Required sum} &= 3 + (2+5) \\ &= 3+7 \\ &= 10 \end{aligned}$$

Again, note that

$$3+2+5 = 10$$

Hence,

$$3+(2+5) = 3+2+5$$

The sum of 2 and 5 is a number and hence $2+5$ has been kept in bracket. Single positive number is not kept in bracket.

Thus you find that if there is '+' sign before the bracket, there is no change in signs of any number when the bracket is removed. Here it appears that bracket has no importance.

Further examples are

$$(i) 5 + (2 - 3) = 5 + 2 - 3 = 7 - 3 = 4$$

$$(ii) 3 + (-5 + 1) = 3 - 5 + 1 = -2 + 1 = -1.$$

Now, note the second example.

Example 2. Subtract sum of 6 and 2 from 10.

$$\begin{aligned} \text{Required difference} &= 10 - (6 + 2) \\ &= 10 - 8 = 2. \end{aligned}$$

$$\text{Again, } 10 - 6 - 2 = 2.$$

Thus we may write—

$$10 - (6 + 2) = 10 - 6 - 2.$$

Note that both the numbers inside bracket are positive and there is '-' sign outside the bracket. See that on removing the bracket both of them have negative sign.

See another example.

Example 3. Subtract the difference of 10 and 4 from 15.

$$\begin{aligned} \text{Required difference} &= 15 - (10 - 4) \\ &= 15 - 6 = 9. \end{aligned}$$

$$\begin{aligned} \text{Again, } 15 - 10 + 4 &= 5 + 4 \quad (\text{From distributive} \\ &= 9 \quad \text{law of addition}). \end{aligned}$$

Thus, we may write—

$$15 - (10 - 4) = 15 - 10 + 4.$$

Note that inside the bracket 10 has positive sign and 4 has negative sign; but after removing bracket 10 has negative sign and 4 has positive sign.

From examples (2) and (3) you find that if there is ‘—’ sign before the bracket, on removing the bracket the sign of each term inside the bracket will be opposite.

Further examples are:

$$(i) \quad 15 - (2 - 4) = 15 - 2 + 4 = 13 + 4 = 17$$

$$(ii) \quad 5 - (8 - 10) = 5 - 8 + 10 = -3 + 10 = 7$$

$$(iii) \quad a - (b - c) = a - b + c$$

You know that we generally use three types of brackets, for example

(1) First bracket or, parenthesis (),

(2) Second bracket or, braces { },

(3) Third bracket or crochets [].

In case of necessity another type of bracket is also used. This is called line bracket or, vinculum ‘—’.

As in arithmetic, in algebra also there may be more than one type of brackets in one expression, for example

$$(1) \quad a + [b - \{c + (d - a)\}]$$

$$(2) \quad a - [x + \{x - (a - \overline{x - a})\}]$$

As in arithmetic, here also the line bracket, first bracket, second bracket and third bracket are removed in the stated order after finding values inside the corresponding brackets.

Example 1. Prove on the number-axis that $4 + 2 = 2 + 4$.

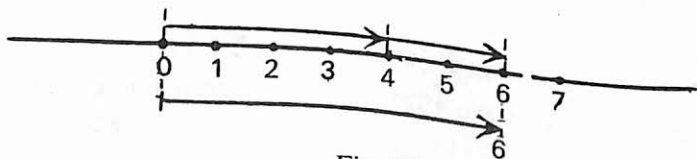


Fig. 20

Adding on the number-axis (Fig. 20), we find that $4+2=6$

Similarly, adding on the number-axis we will find that

$$2+4=6$$

Therefore, $4+2=2+4$

Example 2. Prove that $2 \times 3 = 3 \times 2$.

We know $2 \times 3 = 3+3=6$

Again, $3 \times 2 = 2+2+2=6$

Therefore, $2 \times 3 = 3 \times 2$

Example 3. Simplify: (i) $a(a+2)$
(ii) $(b+5) \times b^2$
(iii) $ab(a+b)$

$$\begin{aligned} (i) \quad a(a+2) &= a \times a + a \times 2 \quad (\text{from distributive law of multiplication}) \\ &= a^2 + 2 \times a \quad (\text{from associative law of multiplication}) \\ &= a^2 + 2a. \end{aligned}$$

$$\begin{aligned} (ii) \quad (b+5)b^2 &= b \times b^2 + 5 \times b^2 \\ &= b^{1+2} + 5b^2 = b^3 + 5b^2 \end{aligned}$$

$$\begin{aligned} (iii) \quad ab(a+b) &= ab \times a + ab \times b \\ &= ba \times a + ab^2 = ba^2 + ab^2 \\ &= a^2b + ab^2 \end{aligned}$$

Example 4. Simplify:

$$-2 + [3 - \{2 - (8 - 3)\}]$$

$$\begin{aligned} \text{Given expression} &= -2 + [3 - \{2 - 5\}] \\ &= -2 + [3 - (-3)] = -2 + [3 + 3] \\ &= -2 + 6 = 4 \end{aligned}$$

Example 5. Simplify:

$$\begin{aligned}
 1 - [3 + 2 - \{8 - (6 - \overline{3 - 5})\}] \\
 \text{iven expression} &= 1 - [3 + 2 - \{8 - (6 + 2)\}] \\
 &= 1 - [3 + 2 - \{8 - 8\}] \\
 &= 1 - [3 + 2 - 0] = 1 - 5 = -4
 \end{aligned}$$

Exercise 9

1. Show on the number-axis:

(a) $1 + 3 = 3 + 1$, (b) $2 + 3 = 3 + 2$.

2. Prove: (i) $3 \times 5 = 5 \times 3$
(ii) $2 \times 4 = 4 \times 2$.

3. Simplify: (i) $a(a+6)$, (ii) $6a(2a+3)$,
(iii) $a^2b(a+b)$, (iv) $abc(a+b+c)$

4. Simplify: (i) $-3 + [2 - \{-5 - (8 - 9)\}]$
(ii) $5 - [3 - 1 - \{2 - (5 + 3 - \overline{2})\}]$
(iii) $-1 - [-1 - \{-1 - (-1 - 1)\}]$
(iv) $5 + 3 - [-8 + \{4 - (-3 - \overline{2 - 4})\}]$

Chapter 4

Polynomials

4.1 Sum:

You have already learnt that if a number of algebraic terms are put together with positive and negative signs between them, then the sum is called an expression. Examples of algebraic expression are as follows:

$$2x+3y$$

$$4a+5b+7c$$

$$a-b+2b \times 3c, \text{ etc.}$$

In an expression the parts which are separated by '+' or '-' sign are called terms. For example, each of $4a$, $5b$, $-b$, $2b \times 3c$, etc. is a term. You have also learnt what is called a co-efficient. For example,

In the term $4a$ the co-efficient of $a=4$

In the term $5b$ the co-efficient of $b=5$

In $-b$ the co-efficient of $b=-1$

In $2b \times 3c=6bc$ the co-efficient of $bc=6$, etc.

The idea of co-efficient has some speciality. For example, in the term $4a$ the co-efficient of a is 4.

But $4a=4 \times a=a+a+a+a$.

This means that in $4a$, a is repeated 4 times.

You have also learnt earlier that if there is one term in an expression it is called monomial, if there are two or three terms in an expression it is called

binomial or trinomial. If there are a number of terms in an expression it is called polynomial.

Example: $2a$: monomial

$2a+3b$: binomial

$3a-5b+7c$: trinomial

$x-2y+5z+8m$: polynomial

Like and unlike terms:

Note the two expressions $xy-2y+4z$ and $3xy+8y+5z$. Both of them are trinomials and there is some similarity between their terms.

For example, in the terms xy and $3xy$ only the co-efficients are different, but the algebraic symbols are same. Same type of terms, of which only the co-efficients are different but the algebraic symbols are same are called like terms. Terms other than like terms are called unlike terms.

Similarly, $-2y$ and $8y$ are like terms,
 $4z$ and $5z$ are like terms.

But xy and $-2y$ are unlike terms,
 $-2y$ and $4z$ are unlike terms, etc.

You know, $2a=a+a$

again $3a=a+a+a$

Therefore, $2a+3a=(a+a)+(a+a+a)$
 $=a+a+a+a+a$
 $=5a.$

Here $2a$ and $3a$ are two like terms and their sum is $5a$. Note that the co-efficient of the sum is the sum of the co-efficients of the two terms which are added.

Similarly:

$$\begin{aligned} 3x+4x &= (x+x+x) + (x+x+x+x) \\ &= x+x+x+x+x+x+x \end{aligned}$$

That is $3x+4x=7x$

Here, $7=3+4$

$$\begin{aligned} \text{Again, } (-3x)+4x &= (-x-x-x) + (x+x+x+x) \\ &= (x-x) + (x-x) + (x-x) + x \\ &= 0+0+0+x=x \end{aligned}$$

Co-efficient of x is 1, see $1=-3+4$

Again see,

$$\begin{aligned} (-y)+(-3y)+(-2y) &= (-y)+(-y-y-y)+(-y-y) \\ &= -y-y-y-y-y-y \\ &= -6y \end{aligned}$$

Again see, $-6=(-1)+(-3)+(-2)$

Thus to add a number of like terms the co-efficient of the sum is obtained by adding the numerical co-efficients of the like terms and the algebraic symbol part will remain same.

If two terms are unlike, their sum is written by putting only '+' sign between them.

For example,

sum of $2a$ and $3b=2a+3b$

sum of x and $y=x+y$

sum of m and $(-n)=m+(-n)=m-n$, etc.

If some of the terms are like terms and some are unlike terms, their sum is obtained by adding the like terms and putting the unlike terms side by side with their respective signs.

For example, the sum of $2a+3b$ and a is

$$2a+3b+a=(2a+a)+3b=3a+3b.$$

To find the sum of two or more expressions, the expressions are written side by side with '+' sign between each pair or they are written one below the other and then added. In the latter method one expression is to be written below another in such a way that the like terms are written in the same column.

Example 1. Add $2x+5$ and $3x-8$.

$$\begin{aligned}\text{Required sum} &= (2x+5)+(3x-8) \\ &= (2x+3x)+5-8 \\ &= 5x-3.\end{aligned}$$

Second Method :

$$\begin{array}{r} 2x+5 \\ 3x-8 \\ \hline \end{array}$$

$$\text{Required sum} = 5x-3$$

Example 2. Add $x+2y-3$, $3x+2y+1$ and $x+y$.

$$\begin{aligned}\text{Required sum} &= (x+2y-3)+(3x+2y+1)+(x+y) \\ &= (x+3x+x)+(2y+2y+y)-3+1 \\ &= 5x+5y-2.\end{aligned}$$

Second Method:

$$\begin{array}{r} x+2y-3 \\ 3x+2y+1 \\ x+y \\ \hline \end{array}$$

$$\text{Required sum} = 5x+5y-2.$$

Example 3. Find the sum of $3x^2+5x+2$, x^2+2 and $x+8$.

First Method:

$$\begin{aligned}\text{Required sum} &= (3x^2+5x+2) + (x^2+2) + (x+8) \\ &= (3x^2+x^2) + (5x+x) + (2+2+8) \\ &= 4x^2+6x+12\end{aligned}$$

Second Method:

$$\begin{array}{r} 3x^2+5x+2 \\ x^2+2 \\ x+8 \\ \hline \end{array}$$

Required sum $= 4x^2+6x+12$

Example 4. Simplify:

$$\begin{aligned}(i) \quad & a^2+5b^2+2a^2-3b+a-8b^2+5a \\ (ii) \quad & x^2y+xy^2+x^3-5x^2y+10xy^2-3x^3\end{aligned}$$

Ans:

(i) Given expression

$$\begin{aligned}&= a^2+2a^2+5b^2-8b^2+a+5a-3b \\ &= (a^2+2a^2) + (5b^2-8b^2) + (a+5a) - 3b \\ &= 3a^2 + (-3b^2) + 6a - 3b \\ &= 3a^2 - 3b^2 + 6a - 3b\end{aligned}$$

(ii) Given expression

$$\begin{aligned}&= x^2y+xy^2+x^3-5x^2y+10xy^2-3x^3 \\ &= x^2y-5x^2y+xy^2+10xy^2+x^3-3x^3 \\ &= (x^2y-5x^2y) + (xy^2+10xy^2) + (x^3-3x^3) \\ &= (-4x^2y) + 11xy^2 + (-2x^3) \\ &= -4x^2y + 11xy^2 - 2x^3.\end{aligned}$$

Exercise 10

1. Add:

- (i) $2a, 3a$ (ii) $7x, 2x, 3x$ (iii) $abc, 2abc$
 (iv) $a^2b, 3a^2b$ (v) $x^2, 5x^2, 6x^2$ (vi) $-x, 8x$
 (vii) $-2x, -3x, -5x$ (viii) $x^2, 5x^2, y^2$
 (ix) $-5x^2, 7x^2, 3y^2$ (x) $xy^2, 2xy^2, -3xy^2$

2. Add:

- (i) $a+2b+c, b+c, a+b-c$
 (ii) $2x+3y+5z, x-y, 2x+8y-z$
 (iii) $x, x+7y-8z, -2x+5z$
 (iv) $5x^2+7x+2, 8x^2-5x+2, 2x^2-2x+1$
 (v) $m^2-mn+p^2, 3m^2+4p^2, 8m^2-9mn+p^2$
 (vi) $x^2+ax+a^2, -x^2-ax+a^2, -2a^2$
 (vii) $8x^3+5xy+7y^2, 6x^3+2xy+9y^2, x^3-y^3$ and x^3+y^3
 (viii) $abc+abc^2+a^2bc, 2abc-3abc^2-a^2bc$ and $-3abc+abc^2+a^2bc$

3. Simplify:

- (i) $3a+(-2a)+5b+(-2b)+8a$
 (ii) $(a-b)+(b+c)+(b-c)+(c-a)$
 (iii) $(m^2+mn+n^2)+(mn-m^2-n^2)+2mn$
 (iv) $(a^2+b^2-c^2)+(b^2+c^2-a^2)$

4.2 Subtraction:

You have seen that to subtract a second number from a first number we are to add the opposite of the second number to the first number. For example subtracting 4 from 10 the difference will be

$$10+(-4)=10-4=6$$

In the same way if b is subtracted from a , the difference will be

$$a + (-b) = a - b.$$

Generally we write as $a - b$ instead of writing as $a + (-b)$. For example,

difference between 4 and $a = 4 - a$

„ „ a and $4 = a - 4$

„ „ x and $y = x - y$, etc.

Thus you see that if from a number a second number is subtracted we write the first number then put a ‘-’ sign and then write the second number.

Subtraction of polynomials:

The above method is also applied to find the difference of two expressions having more than one term.

We first write one expression then put ‘-’ sign and write the expression to be subtracted in a bracket. If we simplify by removing the bracket, we get the difference. Note the following examples carefully.

Example 1. Subtract $x - y$ from $2x + 3y$.

$$\begin{aligned} \text{Required difference} &= (2x + 3y) - (x - y) \\ &= 2x + 3y - x + y \\ &= (2x - x) + (3y + y) \\ &= x + 4y \end{aligned}$$

Example 2. Subtract $2a + 3b - 5c$ from $a + b - c$.

Required difference

$$\begin{aligned}
 &= a + b - c - (2a + 3b - 5c) \\
 &= a + b - c - 2a - 3b + 5c \\
 &= (a - 2a) + (b - 3b) + (-c + 5c) \\
 &= (-a) + (-2b) + 4c \\
 &= -a - 2b + 4c
 \end{aligned}$$

The first expression, from which the second expression is subtracted can be written without bracket.

Example 3. Subtract $x + b - 1$ from $a + 5$.

$$\begin{aligned}
 \text{Required difference} &= a + 5 - (x + b - 1) \\
 &= a + 5 - x - b + 1 \\
 &= a - b - x + (5 + 1) \\
 &= a - b - x + 6.
 \end{aligned}$$

Second method of subtraction:

Like addition we can also subtract by putting like terms in the same column.

The expression from which we are to subtract is written first and below that the expression to be subtracted is written in such a way that the like terms of the two expressions are in the same column. Then the signs of the terms of the lower expression are changed and the difference is obtained by adding these terms with changed signs to those of the first expression.

Example 1. Subtract $x - y$ from $4x + y$.

$$\begin{array}{r}
 4x + y \\
 x - y \\
 \hline
 \ominus \quad \oplus
 \end{array}$$

$$\text{Difference} = 3x + 2y$$

Example 2. Subtract $2a+5b$ from $a+b-c$.

$$\begin{array}{r} a + b - c \\ 2a + 5b \\ \oplus \quad \oplus \\ \hline \end{array}$$

Difference $= -a - 4b - c$

Example 3. Subtract $3m+2n-4p$ from $m-n$.

$$\begin{array}{r} m - n \\ 3m + 2n - 4p \\ \oplus \quad \oplus \quad \oplus \\ \hline \end{array}$$

Difference $= -2m - 3n + 4p$

At the time of learning subtraction you may write the changed sign below the second expression and then add. But afterwards you should practise to change the signs mentally without actually writing them and then to add.

Exercise 11

1. Subtract second from the first:

- (i) $a, 2a$ (ii) $-3a, 4a$ (iii) $a^2, -5a^2$
 (iv) $abc, 2abc$ (v) $ab^2, -3ab^2$.

2. Subtract:

- (i) $-2a+3b-c$ from $a+b-c$
 (ii) $a^2+b^2+c^2$ from $3a^2-b^2+c^2$
 (iii) $2x+5$ from $5x^2+8x+5$
 (iv) $3x^2+5$ from $6x^2+7x$
 (v) $2ab^2-3$ from $ab^2+5ab+3$

3. What is to be added to x^2+8x+5 to get x^2 ?

4. What is to be added to $m^2+n^2+p^2$ to get m^2+p^2 ?

4.3 Easy multiplication :

You have already learnt about the sign; commutative, associative and distributive laws; and law of indices of multiplication. We will now discuss how to find the product of algebraic expressions by applying those laws.

Multiplication of monomial by monomial:

It is necessary to put '×' sign between the expressions which are being multiplied. Multiplication is performed by separately multiplying the co-efficients and applying the law of indices to the symbols. See the following examples.

Example 1. Multiply :

(i) $4xy$ by $2x^2y$

(ii) $3a^2b$ by $-2a$

(iii) $-5mn^2$ by $-4n^3$.

$$\begin{aligned} \text{(i)} \quad 4xy \times 2x^2y &= 4 \times x \times y \times 2 \times x^2 \times y \\ &= (4 \times 2) \times (x \times x^2) \times (y \times y) \\ &= 8x^3y^2 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 3a^2b \times (-2a) &= 3 \times a^2 \times b \times (-2) \times a \\ &= 3 \times (-2) \times (a^2 \times a) \times b \\ &= -6a^3b \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad -5mn^2 \times (-4n^3) &= (-5) \times m \times n^2 \times (-4) \times n^3 \\ &= (-5) \times (-4) \times m \times (n^2 \times n^3) \\ &= 20mn^5 \end{aligned}$$

Example 2. Simplify:

$$(2a^2b) \times (-4ab^2) \times (-8a^2b^2)$$

$$\begin{aligned}\text{Here } 2a^2b \times (-4ab^2) &= 2 \times (-4) \times (a^2 \times a) \times (b \times b^2) \\ &= -8a^3b^3\end{aligned}$$

Thus, given expression

$$\begin{aligned}&= -8a^3b^3 \times (-8a^2b^2) \\ &= (-8) \times (-8) \times (a^3 \times a^2) \times (b^3 \times b^2) \\ &= 64a^5b^5\end{aligned}$$

Example 3. Multiply:

$$mn^2 \times (-m^3n) \times (-3m^2n) \times 5mn^4$$

Given expression

$$\begin{aligned}&= (-1)(-3) \times 5 \times m \times m^3 \times m^2 \times m \times n^2 \times n \times n \times n^4 \\ &= 15m^{1+3+2+1} \times n^{2+1+1+4} \\ &= 15m^7n^8\end{aligned}$$

[Numerical co-efficient of $m^3n = -1$

„ „ „ $m^2n = -3$

„ „ „ $mn^2 = +5$

These co-efficients are first separated and put together. The co-efficient of mn^2 is 1 which we may not write.]

Exercise 12

Multiply the first expression by the second:

- | | | |
|----------------------------|---------------------------------|-----------------------|
| 1. $2a^2, 3a$ | 2. $3x^2, -3x^3$ | 3. $8m^2, 3m^5$ |
| 4. $-2ab, 3a^2b^3$ | 5. $-5, 3ab$ | 6. $7a^3b^2, -8a^5bc$ |
| 7. $9a^3b^2c, -3a^5b^6c^7$ | 8. $-10a^3b^3c^3, -3ab^2c^5$ | |
| 9. $-5p^3q^3r, -6pq^5r^4$ | 10. $-15m^3n^3t^5, -5m^8n^5t^3$ | |

Find the product:

- | | |
|---|---|
| 11. $-3a \times 2b \times 5c$ | 12. $8a^3 \times (-3b^3) \times 5a^2$ |
| 13. $-ab^2 \times 2a^2b^3 \times (-3ab)$ | 14. $\frac{1}{2}a^3 \times \frac{-3}{4}a^2 \times \frac{8}{9}a^6$ |
| 15. $-3ab \times (-2bc) \times 5ca$ | 16. $3a^2b \times (-b^2c) \times 8c^2a$ |
| 17. $abc \times bca \times cab$ | 18. $-ab^3 \times 3ab^2 \times (-8a^3b^2)$ |
| 19. $-3x^2yz \times (-6xy^3z^2) \times (-3x^3y^3z^3)$ | |
| 20. $-2mn^3 \times (-6m^2n^2) \times m^4n^4$ | |

Simplify:

21. $(3abc) \times (5xyz) \times (-2x)$
 22. $(-8a^2bc) \times (-abc^2) \times (-abc)$
 23. $(-3abc) \times (-2ad) \times (2bd) \times (-cd)$
 24. $(-\frac{1}{2}ab^2) \times (-\frac{2}{3}a^3c^3) \times (-3ca^2)$
 25. $(-3p^2q^2r^2) \times (-8r^2s^2t^2) \times (-2p^3q^2r^4)$.
-

Multiplication of binomial by monomial :

You know from the distributive law of multiplication,

$$a(b+c) = ab+ac \quad \dots (1)$$

$$\text{and } (a+b)c = ac+bc \quad \dots (2)$$

According to this law,

$$\begin{aligned} 2a(3a+2) &= 2a.3a + 2a.2 \\ &= 6a^2 + 4a \end{aligned}$$

$$\begin{aligned} 3x(8x^2+5y) &= 3x.8x^2 + 3x.5y \\ &= 24x^3 + 15xy \end{aligned}$$

$$\begin{aligned} \text{Again, } (2a+5)a^2 &= 2a.a^2 + 5.a^2 \\ &= 2a^3 + 5a^2 \end{aligned}$$

$$\begin{aligned} (6m+3n) \times 2m &= 6m \times 2m + 3n \times 2m \\ &= 12m^2 + 6mn \end{aligned}$$

Further examples are :

$$\begin{aligned} (i) \quad 2x(-3x^2+5) &= 2x(-3x^2) + 2x.5 \\ &= -6x^3 + 10x \end{aligned}$$

$$\begin{aligned} (ii) \quad 8y^2(3y-6) &= 8y^2.3y + 8y^2(-6) \\ &= 24y^3 - 48y^2 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (4a+5b)(-3a) &= 4a(-3a) + 5b(-3a) \\ &= -12a^2 - 15ab. \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad (3m^2-2n)(-m^3) &= 3m^2(-m^3) - 2n(-m^3) \\ &= -3m^5 + 2m^3n \end{aligned}$$

Multiplication of binomial by binomial:

You have already seen that

$$(a+b)c = ac + bc$$

Let $c = x + y$, putting the value of c , we get

$$\begin{aligned} (a+b)(x+y) &= a(x+y) + b(x+y) \\ &= ax + ay + bx + by \end{aligned}$$

In the same way,

$$\begin{aligned} (m+n)(p+q) &= m(p+q) + n(p+q) \\ &= mp + mq + np + nq \\ (a+x)(b+y) &= a(b+y) + x(b+y) \\ &= ab + ay + xb + xy \end{aligned}$$

Thus you find that to multiply a binomial by another binomial it is necessary to put both expressions in bracket and put them side by side.

Required product will be as follows:

First term of the first bracket $\times$ whole expression of the second bracket $+$ second term of the first bracket $\times$ whole expression of the second bracket. The rest of the work is done by applying distributive law of multiplication.

In a still shorter way we may say that the product is the successive sum of the product of every term of the first expression with every term of the second.

Instead of putting the two expressions side by side in brackets the product may be found in a different way.

Example 1. Multiply $2a+3b$ by $a+b$.

$$\begin{aligned}\text{Required product} &= (2a+3b)(a+b) \\ &= 2a(a+b) + 3b(a+b) = 2a.a + 2a.b + 3b.a + 3b.b \\ &= 2a^2 + 2ab + 3ab + 3b^2 = 2a^2 + 5ab + 3b^2\end{aligned}$$

Short Method:

$$\begin{aligned}(2a+3b)(a+b) &= 2a.a + 2a.b + 3b.a + 3b.b \\ &= 2a^2 + 2ab + 3ba + 3b^2 = 2a^2 + 5ab + 3b^2\end{aligned}$$

Alternative method:

$$\begin{array}{r} 2a+3b \\ a+b \\ \hline 2a^2+3ab \\ +2ab+3b^2 \\ \hline \end{array} \quad \begin{array}{l} \text{(Put like terms} \\ \text{one below the} \\ \text{other and add)} \end{array}$$

$$\text{Required product} = 2a^2 + 5ab + 3b^2$$

Example 2. Multiply $8x^2+2x$ by $x+5$.

Required product

$$\begin{aligned}&= (8x^2+2x)(x+5) = (x+5)(8x^2+2x) \\ &= x(8x^2+2x) + 5(8x^2+2x) \\ &= x.8x^2 + x.2x + 5.8x^2 + 5.2x \\ &= 8x^3 + 2x^2 + 40x^2 + 10x \\ &= 8x^3 + 42x^2 + 10x\end{aligned}$$

Short method:

$$\begin{aligned}(x+5)(8x^2+2x) &= x.8x^2 + x.2x + 5.8x^2 + 5.2x \\ &= 8x^3 + 2x^2 + 40x^2 + 10x = 8x^3 + 42x^2 + 10x\end{aligned}$$

Alternative method:

$$\begin{array}{r} 8x^2+2x \\ x+5 \\ \hline 8x^3+2x^2 \\ +40x^2+10x \\ \hline \end{array}$$

$$\text{Required product} = 8x^3 + 42x^2 + 10x$$

Example 3. If $a = x^2 - y^2$, $b = y^2 - z^2$, find the value of $a(x+y) + b(y+z)$.

Given expression

$$\begin{aligned} &= (x^2 - y^2)(x+y) + (y^2 - z^2)(y+z) \\ &= x^2(x+y) - y^2(x+y) + y^2(y+z) - z^2(y+z) \\ &= x^3 + x^2y - xy^2 - y^3 + y^3 + y^2z - yz^2 - z^3 \\ &= x^3 + x^2y - xy^2 + y^2z - yz^2 - z^3 \end{aligned}$$

Exercise 13

Multiply :

1. $x+2$ by $3x+1$.
2. $2x-3$ by $6x+8$.
3. x^2+y^2 by $x+y$.
4. $2x^2-a$ by $a+x$.
5. $5a^4+6b$ by $2a+3b$.
6. $8a^2-6a$ by $-a-b$.
7. $9-5a$ by $-3a^2+b$.
8. $11a^2+5a$ by $9a^2-3a$.
9. $3x^2-7y^2$ by $mx+ny$.
10. $2a^3-3a$ by $-11a+7$.
11. $8x^2+7x^4$ by $9x^3+11x^2$.
12. $xy-yz$ by $x-y$.
13. ax^3+by^3 by a^2+b^2 .
14. $8x^3-13y^2$ by $2x^3-11y^2$.
15. ax^2+c by $-bx^2+d$.
16. Simplify:

(i) $(a+b)(a-b) + (b+c)(b-c) + (c+a)(c-a)$

(ii) $(a^2+b^2)(a^2-b^2) + (b^2+c^2)(b^2-c^2) + (c^2+a^2)(c^2-a^2)$

(iii) $(a+b)(a^2-b^2) + (b+c)(b^2-c^2) + (c+a)(c^2-a^2)$

17. If $a = lx+m$, $b = my+n$, $c = nz+l$, find the value of $a(l+m) + b(m+n) + c(n+l)$.

18. If $p=ax+by$, $q=by+cz$ and $r=cz+ax$; find the value of $p(a+b)+q(b+c)+r(c+a)$.
-

4.4 Easy division:

Division of a polynomial by monomial
You have already seen,

$$(a+b) \div x = \frac{a+b}{x} = \frac{a}{x} + \frac{b}{x}.$$

Similarly,

$$(a+b+c) \div x = \frac{a+b+c}{x} = \frac{a}{x} + \frac{b+c}{x} \\ = \frac{a}{x} + \frac{b}{x} + \frac{c}{x}$$

$$(a+b+c+d) \div x = \frac{a+b+c+d}{x} = \frac{a}{x} + \frac{b}{x} + \frac{c}{x} + \frac{d}{x}.$$

That is, required quotient is the sum of the quotients obtained by dividing each term of the dividend by the divisor.

For example,

$$(1) \quad \frac{a+b+c}{5} = \frac{a}{5} + \frac{b}{5} + \frac{c}{5}$$

$$(2) \quad \frac{2a+3b+5c}{x} = \frac{2a}{x} + \frac{3b}{x} + \frac{5c}{x}.$$

Index rule of division.

From the index rule of multiplication, you know

$$a^m \times a^n = a^{m+n}$$

Hence, from the idea of division

$$a^{m+n} \div a^m = a^n$$

$$\text{That is } \frac{a^{m+n}}{a^m} = a^n = a^{(m+n)-m}$$

Suppose, $m+n=p$

Then we have

$$a^p \div a^m = \frac{a^p}{a^m} = a^{p-m}$$

Thus $\frac{a^7}{a^3} = a^{7-3} = a^4,$

$$\frac{x^5}{x^2} = x^{5-2} = x^3, \text{ etc.}$$

We can consider the above examples in a different way. For example,

Dividend	$= a^7$	= successive multiplication of 7 a's
Divisor	$= a^3 =$	„ „ „ 3 a's
Quotient	$= a^4 =$	„ „ „ 4 a's

Thus we find that if we omit 3a's of the divisor from 7a's of the dividend, the product of rest 4a's will be the required quotient.

In the same manner, we have

$$(1) \quad \frac{x^2}{x} = x;$$

$$(2) \quad \frac{x^3}{x^2} = x$$

$$(3) \quad \frac{x^7}{x^2} = x^5;$$

$$(4) \quad \frac{a^2 b}{a} = ab$$

$$(5) \quad \frac{a^3 b^3}{ab} = a^2 b^2;$$

$$(6) \quad \frac{a^5 x^2}{ax} = a^4 x$$

Note: $\frac{a}{a} = 1, \quad \frac{a^2}{a^2} = 1, \quad \frac{x^3}{x^3} = 1.$

You will understand these easily as the quotient of dividing a number by itself is always 1.

We have already discussed the rules of signs for division.

Corollary: From the index rule of division, $a^{m-n} = a^m \div a^n$. Now if $m=n$, $a^{n-n} = a^n \div a^n = 1$. That is $a^0 = 1 (a \neq 0)$. Thus the value of any numerical symbol to the power '0' = 1.

Example 1. Divide $3a^4 + 5a^3 + 2a^2$ by a^2 .

$$\begin{aligned}\text{Required quotient} &= \frac{3a^4 + 5a^3 + 2a^2}{a^2} \\ &= \frac{3a^4}{a^2} + \frac{5a^3}{a^2} + \frac{2a^2}{a^2} \\ &= 3a^2 + 5a + 2\end{aligned}$$

Example 2. Find the value of

$$\frac{30a^2b^3 + 40a^3b^4 - 50a^4b^5}{10a^2b^2}$$

$$\begin{aligned}\text{Required value} &= \frac{30a^2b^3}{10a^2b^2} + \frac{40a^3b^4}{10a^2b^2} - \frac{50a^4b^5}{10a^2b^2} \\ &= 3b + 4ab^2 - 5a^2b^3.\end{aligned}$$

Exercise 14

Divide:

- $6ab - 3a^2$ by $3a$.
- $8a^3 + 10a^2 + 6a$ by $2a$.
- $-15a^2b^3 + 10a^3b^4 + 5a^4b^3$ by $-5a^2b^2$.
- $8a^5x^5 - 4a^4x^4 + 4a^3x^3$ by $-4a^2x^2$.
- $4m^4p^3 - 4m^3p^4 + 4m^2p^4$ by $-2mp$.
- $9p^4q^4 - 27p^3q^3 + 18p^2q^2$ by $3pq^2$.
- $3a^3b^3c^3 + 5a^2b^4c + 8ab^3c^4$ by $-12abc$.
- $12a^2b^3 - 20a^3b^2 + 48a^3b^3$ by $16a^2b^2$.
- $m^2n^4 + m^3n^2 - m^2n^3 + m^4n^2$ by $-3m^2n^2$.
- $8p^4m^4 + 12p^5m^2 - 16m^2p^3$ by $8m^2p^3$.
- $3pqr + 8p^2qr^2 - 9p^3q^2r^2$ by $-3pqr$.

Chapter 5

Algebraic Formulae and their easy applications

5.1 Square of the sum of two terms:

You know that if an expression is multiplied by itself then the product is the square of the expression. You have seen in arithmetic, $5 \times 5 = 25$ or, $5^2 = 25$, $9 \times 9 = 81$ or, $9^2 = 81$, $11 \times 11 = 121$ or, $11^2 = 121$; here 25, 81, 121 are the squares of 5, 9, 11 respectively. Similarly, in algebra also we can write $a^2 = a \times a$, $x^2 = x \times x$, $p^2 = p \times p$, etc. Now suppose that any expression x is the sum of two terms a and b .

That is $x = a + b$

Now $x^2 = x \times x$

$$\therefore (a+b)^2 = (a+b)(a+b)$$

Now let us see what is the result of multiplying $(a+b)$ by $(a+b)$.

$$(a+b)^2 = (a+b)(a+b)$$

$$= a(a+b) + b(a+b)$$

$$= a^2 + ab + ba + b^2$$

$$= a^2 + ab + ab + b^2$$

$$= a^2 + 2ab + b^2$$

[by commutative law
of multiplication

$$ab = ba]$$

By ordinary multiplication:

$$\begin{array}{r} a+b \\ a+b \\ \hline \end{array}$$

$$a^2 + ab$$

$$ab + b^2$$

$$\hline a^2 + 2ab + b^2$$

In like manner, multiplying $(x+y)$ by $(x+y)$ or $(m+n)$ by $(m+n)$, we will find that

$$\begin{array}{l|l}
 (x+y)^2 = (x+y)(x+y) & (m+n)^2 = (m+n)(m+n) \\
 = x(x+y) + y(x+y) & = m(m+n) + n(m+n) \\
 = x^2 + xy + xy + y^2 & = m^2 + mn + mn + n^2 \\
 = x^2 + 2xy + y^2 & = m^2 + 2mn + n^2
 \end{array}$$

Each of the above products is the square of the sum of two terms. Note that each product has a general form. We can write that general form in the following manner:

$$(\text{1st term} + 2\text{nd term})^2 = (\text{1st term})^2 + 2(\text{1st term} \times 2\text{nd term}) + (\text{2nd term})^2.$$

Now denoting the 1st and the 2nd terms by the symbols a and b respectively, the above rule may be written as

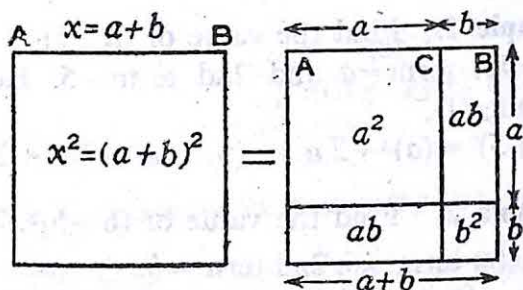
$$(a+b)^2 = a^2 + 2ab + b^2 \quad \dots \quad (I)$$

The expression in the right hand side is called the expansion of $(a+b)^2$, that is finding square of $(a+b)$ means finding the expansion of $(a+b)^2$. We can use this rule in finding the square of the sum of any two terms. This is an important formula of algebra.

The truth of this formula for finding the square of the sum of two terms can also be verified by geometric method.

In the figure of page 77, length of the line segment AB is $x = a + b$. Length of the line segment AC = a . Length of the line segment BC = b . Area of the square drawn on AB = $x^2 = (a+b)^2$.

Area of the square on $AC=a^2$ and that on $BC=b^2$.



1st figure

2nd figure

$$(a+b)^2 = a^2 + 2ab + b^2$$

▲ the square drawn on $(a+b)$ [2nd fig.] is composed of the square drawn on a , that drawn on b and two more rectangles with sides a and b —giving the area of each rectangle as ab . Thus from the geometric figure we get,

$$(a+b)^2 = a^2 + 2ab + b^2.$$

Let us now test the above formula by taking an example. You know $15^2 = 225$. We get this value by multiplying 15 straight away by 15. Now we apply the formula. Write 15 as the sum of two terms in the form $10+5$.

$$\begin{aligned}
 \text{▲ } (15)^2 &= (10+5)^2 \\
 &= (a+b)^2 \text{ [taking } 10=a, 5=b\text{]} \\
 &= a^2 + 2ab + b^2 \\
 &= (10)^2 + 2 \cdot 10 \cdot 5 + (5)^2 \text{ [putting values of } a \text{ and } b\text{]} \\
 &= 100 + 100 + 25 = 225
 \end{aligned}$$

Thus we get the same value both by straight multiplication and by the help of formula.

You can take some more numbers and verify in the above manner.

Example 1. Find the value of $(a+5)^2$.

Here 1st term = a and 2nd term = 5 . Hence by the formula (I),

$$(a+5)^2 = (a)^2 + 2.a.5 + (5)^2 = a^2 + 10a + 25.$$

Example 2. Find the value of $(6+b)^2$.

Here, 1st term = 6 , 2nd term = b .

$$\therefore \text{by the formula, } (6+b)^2 = (6)^2 + 2.6.b + (b)^2 \\ = 36 + 12b + b^2.$$

Example 3. Find the square of 902.

$$(902)^2 = (900+2)^2 = (900)^2 + 2.900.2 + (2)^2 \\ = 810000 + 3600 + 4 \\ = 813604.$$

You know, $2a \times 2a = 4a^2$

$$\text{or, } (2a)^2 = 2^2.a^2$$

Thus if any algebraic term has a co-efficient, the square of the term will be the product of the square of the co-efficient and that of the algebraic symbol.

In this way, $5m \times 5m = 5^2.m^2$

Therefore, $(5m)^2 = 5^2.m^2 = 25m^2$.

Similarly, $(ab)^2 = a^2b^2$

$$(abc)^2 = a^2b^2c^2.$$

Example 4. Find the square of $3p+q$.

$$(3p+q)^2 = (a+b)^2 \text{ [taking } 3p=a, q=b] \\ = a^2 + 2ab + b^2$$

$$= (3p)^2 + 2.3p.q + q^2 \text{ [putting values of } a \text{ and } b] \\ = 9p^2 + 6pq + q^2.$$

Example 5. Find the value of $(4m+5n)^2$.

$$\begin{aligned}
 (4m+5n)^2 &= (a+b)^2 && [\text{taking } 4m=a, 5n=b] \\
 &= a^2 + 2ab + b^2 \\
 &= (4m)^2 + 2 \cdot 4m \cdot 5n + (5n)^2 \\
 &&& [\text{putting values of } a \text{ and } b] \\
 &= 16m^2 + 40mn + 25n^2
 \end{aligned}$$

Square of the sum of three or more terms:

With the help of the formula for the square of the sum of two terms we may find the square of the sum of three or more terms. See the following two examples.

Example 6. Find the value of $(x+y+z)^2$.

$$\begin{aligned}
 (x+y+z)^2 &= \{(x+y)+z\}^2 \\
 &= (a+b)^2 && [\text{taking } x+y=a, z=b] \\
 &= a^2 + 2ab + b^2 \\
 &= (x+y)^2 + 2(x+y)z + z^2 \\
 &&& [\text{putting the values of } a, b] \\
 &= x^2 + 2xy + y^2 + 2zx + 2yz + z^2 \\
 &&& [\text{applying the formula again}] \\
 &= x^2 + y^2 + z^2 + 2yz + 2zx + 2xy
 \end{aligned}$$

Example 7. Find the value of $(p+q+r+s)^2$.

$$\begin{aligned}
 (p+q+r+s)^2 &= \{(p+q)+(r+s)\}^2 \\
 &= (a+b)^2 && [\text{taking } p+q=a, r+s=b] \\
 &= a^2 + 2ab + b^2 \\
 &= (p+q)^2 + 2(p+q)(r+s) + (r+s)^2 \\
 &= p^2 + 2pq + q^2 + 2pr + 2qr + 2ps + 2qs \\
 &&& + r^2 + 2rs + s^2 \\
 &= p^2 + q^2 + r^2 + s^2 + 2pq + 2pr + 2ps \\
 &&& + 2qr + 2qs + 2rs.
 \end{aligned}$$

The expansion of Example 6 is also used as a formula for the square of the sum of three terms.

Exercise 15

Find the square of the following expressions:

- | | | | |
|------------------|-----------------|---------------|--------------|
| 1. $a+3$ | 2. $8+b$ | 3. $1+c$ | 4. $a+x$ |
| 5. $m+p$ | 6. $6x+1$ | 7. $3a+5$ | 8. $1+9c$ |
| 9. $m+2n$ | 10. $a+3b$ | 11. $p+4q$ | 12. $4x+5y$ |
| 13. $8a+3b$ | 14. $5c+7d$ | 15. $10m+5n$ | 16. $ab+c$ |
| 17. $p+qr$ | 18. $xy+2z$ | 19. $3a+bc$ | 20. $lm+np$ |
| 21. $2xy+z$ | 22. $a+3bc$ | 23. $abc+d$ | 24. $3x+mnp$ |
| 25. $a+b+8$ | 26. $m+n+p$ | 27. $a+x+2p$ | 28. $x+2y+z$ |
| 29. $3m+2n+p$ | 30. $a+b+cd$ | 31. $x+2y+zp$ | |
| 32. $p+q+r+5$ | 33. $l+2m+n+3p$ | | |
| 34. $1+2a+3b+4c$ | 35. $a+2b+3c+d$ | | |
| 36. $m+n+2p+3q$ | | | |

The algebraic formula which you have just learnt may be used to write some given expressions as perfect squares.

Example 8. Express $16m^2+40mn+25n^2$ as a perfect square.

$$\begin{aligned}
 16m^2+40mn+25n^2 &= (4m)^2 + 2 \cdot 4m \cdot 5n + (5n)^2 \\
 &= a^2 + 2ab + b^2 \\
 &\quad \text{[taking } 4m = a, 5n = b\text{]} \\
 &= (a+b)^2 \\
 &= (4m+5n)^2 \\
 &\quad \text{[putting values of } a \text{ and } b\text{]}
 \end{aligned}$$

Example 9. Express $25x^2+10x+1$ as a perfect square and hence find its value when $x = -\frac{1}{5}$.

$$25x^2 + 10x + 1 = (5x)^2 + 2.5x.1 + (1)^2$$

[The expression has been expressed in the form $a^2 + 2ab + b^2$]

$$= (5x + 1)^2$$

$$= \{5(-\frac{1}{5}) + 1\}^2 \quad [\text{putting } x = -\frac{1}{5}]$$

$$= (-1 + 1)^2 = 0.$$

Example 10. Simplify with the help of formula :

$$(3x + 5y)^2 + 2(3x + 5y)(2x - 5y) + (2x - 5y)^2$$

$$\text{Let } 3x + 5y = a, \quad 2x - 5y = b$$

$$\text{Given expression} = a^2 + 2ab + b^2$$

$$= (a + b)^2$$

$$= (3x + 5y + 2x - 5y)^2$$

[putting values of a and b]

$$= (5x)^2 = 25x^2$$

From the algebraic formula that you have learnt, we may write

$$\begin{aligned} a^2 + b^2 &= a^2 + b^2 + 2ab - 2ab \quad [\text{adding and subtracting } 2ab] \\ &= (a + b)^2 - 2ab \end{aligned}$$

Example 11. If $a + b = 5$, $ab = 6$; what is the value of $a^2 + b^2$?

$$\begin{aligned} \text{Now, } a^2 + b^2 &= (a + b)^2 - 2ab \\ &= (5)^2 - 2.6 = 25 - 12 = 13 \end{aligned}$$

$\therefore$ the value of $a^2 + b^2$ is 13.

Example 12. If $x + \frac{1}{x} = 8$, prove that $x^2 + \frac{1}{x^2} = 62$.

$$\begin{aligned} x^2 + \frac{1}{x^2} &= a^2 + b^2 \left[\text{taking } x = a, \frac{1}{x} = b \right] \\ &= (a + b)^2 - 2ab \end{aligned}$$

$$= \left(x + \frac{1}{x}\right)^2 - 2x \cdot \frac{1}{x} \text{ [putting values of } a, b\text{]} \\ = (8)^2 - 2 = 64 - 2 = 62 \text{ [proved].}$$

[Note: Square of a fraction is equal to the quotient of the squares of its numerator and denominator.

$$\text{That is } \left(\frac{a}{b}\right)^2 = \frac{(a)^2}{(b)^2} = \frac{a^2}{b^2} \text{ and } \left(\frac{1}{x}\right)^2 = \frac{(1)^2}{(x)^2} = \frac{1}{x^2}.$$

Exercise 16

Simplify with the help of formula:

1. $(m+n)^2 + 2(m+n)(m-n) + (m-n)^2$
2. $(5x+y)^2 + 2(5x+y)(x-y) + (x-y)^2$
3. $(3a-4b)^2 + 2(3a-4b)(a+4b) + (a+4b)^2$
4. $(3a-2b+c)^2 + 2(3a-2b+c)(a+2b-c) + (a+2b-c)^2$
5. $(3x+2y-4z)^2 + 2(3x+2y-4z)(4z-2x-y) + (4z-2x-y)^2$

Express as a perfect square:

- | | |
|----------------------------|--------------------------------------|
| 6. $9a^2 + 6a + 1$ | 7. $4a^2 + 2a + \frac{1}{4}$ |
| 8. $4a^2 + 12ab + 9b^2$ | 9. $1 + \frac{4}{a} + \frac{4}{a^2}$ |
| 10. $25a^2 + 40ab + 16b^2$ | |

Find the value after expressing as a perfect square:

11. $131 \times 131 + 2 \times 131 \times 69 + 69 \times 69$
12. $6.67 \times 6.67 + 2 \times 6.67 \times 3.33 + 3.33 \times 3.33$
13. $49a^2 + 14a + 1$ when $a=1$
14. $9x^2 + 24xy + 16y^2$ when $x=3, y=-2$
15. $36m^2 + 132mn + 121n^2$ when $m=8, n=-5$
16. $64a^2 + 16a + 1$ when $a=-\frac{1}{8}$

17. If $a+b=7$, $ab=21$; what is the value of a^2+b^2 ?
18. If $x+y=2$, $xy=-15$; what is the value of x^2+y^2 ?
19. If $x+y=-8$, $xy=-20$, what is the value of x^2+y^2 ?
20. If $m+n=0$, $mn=-25$, what is the value of m^2+n^2 ?
21. If $m+n=9$, $mn=18$; prove that $m^2+n^2=45$.
22. If $p+q=-1$, $pq=-20$; prove that $p^2+q^2=41$.
23. If $a+\frac{1}{a}=5$, find the value of $a^2+\frac{1}{a^2}$.
24. If $p+\frac{1}{p}=7$, find the value of $p^2+\frac{1}{p^2}$.
25. If $x+\frac{1}{x}=6$, prove that $x^2+\frac{1}{x^2}=34$.
26. If $m+\frac{1}{m}=-5$, prove that $m^2+\frac{1}{m^2}=23$.

5.2 Square of the difference of two terms :

You know, $x^2 = x \times x$; now if x is the difference of the two terms a and b , we have $x = a - b$. Thus putting the value of x , $(a-b)^2 = (a-b) \times (a-b)$.

Let us see what we do get by multiplying $(a-b)$ by $(a-b)$.

$$\begin{aligned}
 (a-b)^2 &= (a-b)(a-b) \\
 &= a(a-b) - b(a-b) \\
 &= a^2 - ab - ba + b^2 \\
 &= a^2 - ab - ab + b^2 \\
 &= a^2 - 2ab + b^2
 \end{aligned}$$

By ordinary multiplication:

$$\begin{array}{r}
 a-b \\
 a-b \\
 \hline
 a^2-ab \\
 -ab+b^2 \\
 \hline
 a^2-2ab+b^2
 \end{array}$$

Thus we get

$$(a-b)^2 = a^2 - 2ab + b^2.$$

Similarly multiplying $(p-q)$ by $(p-q)$ or $(c-d)$ by $(c-d)$, we get

$$\begin{array}{l|l} (p-q)^2 = (p-q) \times (p-q) & (c-d)^2 = (c-d) \times (c-d) \\ = p(p-q) - q(p-q) & = c(c-d) - d(c-d) \\ = p^2 - pq - pq + q^2 & = c^2 - cd - cd + d^2 \\ = p^2 - 2pq + q^2 & = c^2 - 2cd + d^2 \end{array}$$

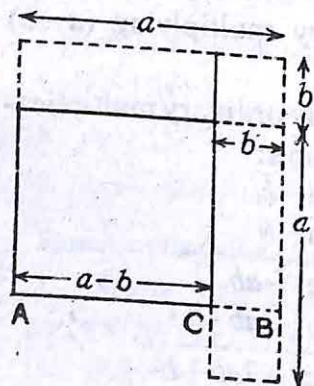
We find that there is a common property in the above products. We can express this common property, as before, in the form of a formula.

$$(1\text{st term} - 2\text{nd term})^2 = (1\text{st term})^2 - 2 \times (1\text{st term} \times 2\text{nd term}) + (2\text{nd term})^2.$$

Taking 1st term as a and 2nd term as b , we can write, $(a-b)^2 = a^2 - 2ab + b^2$ (II)

As before, the expression in the right hand side may be called the expansion of $(a-b)^2$.

As before, we may test geometrically the truth of this formula.



In the adjoining figure length of line segment AB is a , length of line segment BC is b and length of line segment AC is $a-b$.

Area of square on AC = $(a-b)^2$,
area of square on AB = a^2 ,
area of square on BC = b^2 .

From the figure we find that if we subtract two rectangles with sides a and b from the sum of the

squares on a and b we get the square on $a-b$.

Thus from this geometric figure, we get

$$(a-b)^2 = a^2 - 2ab + b^2$$

Hence, we may write, square of the difference of two terms = difference between the sum of the squares of the two terms and twice the product of the two terms.

We can also get this formula from the expansion of $(a+b)^2$.

$$\begin{aligned}(a-b)^2 &= \{a+(-b)\}^2 \\ &= \{a+x\}^2. \text{ [taking } -b=x\text{]} \\ &= a^2 + 2ax + x^2 = a^2 + 2a(-b) + (-b)^2 \\ &= a^2 - 2ab + b^2\end{aligned}$$

Note: $(a+b)^2 = a^2 + b^2 + 2ab$
 $(a-b)^2 = a^2 + b^2 - 2ab$

The only difference between the two expansions is the sign of the term $2ab$. The reason for this is that in the first case both the terms are positive and in the second one is positive and the other is negative.

Example 1. Find the expansion of $(a-4)^2$.

$$(a-4)^2 = (a)^2 - 2.a.4 + (4)^2 = a^2 - 8a + 16.$$

Example 2. Find the expansion of $(5-x)^2$.

$$(5-x)^2 = (5)^2 - 2.5.x + (x)^2 = 25 - 10x + x^2.$$

Example 3. Find the value of $(999)^2$.

$$\begin{aligned}(999)^2 &= (1000-1)^2 = (1000)^2 - 2.1000.1 + (1)^2 \\ &= 1000000 - 2000 + 1 \\ &= 1000001 - 2000 \\ &= 998001\end{aligned}$$

Example 4. Find the square of $4m-n$.

$$\begin{aligned}(4m-n)^2 &= (4m)^2 - 2.4m.n + (n)^2 \\ &\quad [\text{Suppose } 4m=a, n=b] \\ &= 16m^2 - 8mn + n^2\end{aligned}$$

Example 5. Write the expansion of $(6x-5y)^2$.

$$\begin{aligned}(6x-5y)^2 &= (6x)^2 - 2.6x.5y + (5y)^2 \\ &\quad [\text{taking } 6x=a, 5y=b] \\ &= 36x^2 - 60xy + 25y^2\end{aligned}$$

Example 6. Find the value of $(x+y-z)^2$.

$$\begin{aligned}(x+y-z)^2 &= \{(x+y)-z\}^2 \\ &= \{a-z\}^2 \quad [\text{taking } x+y=a] \\ &= a^2 - 2az + z^2 \\ &= (x+y)^2 - 2(x+y)z + z^2 \\ &\quad [\text{putting the value of } a] \\ &= x^2 + 2xy + y^2 - 2zx - 2yz + z^2 \\ &= x^2 + y^2 + z^2 - 2yz - 2zx + 2xy.\end{aligned}$$

Example 7. With the help of the expansion of $(a+b)^2$ find the expansion of $(2x-3y)^2$.

$$\begin{aligned}(2x-3y)^2 &= \{2x+(-3y)\}^2 \\ &= (a+b)^2 \quad [\text{taking } 2x=a, -3y=b] \\ &= a^2 + 2ab + b^2 \\ &= (2x)^2 + 2.2x(-3y) + (-3y)^2 \\ &\quad [\text{putting the values of } a, b] \\ &= 4x^2 - 12xy + 9y^2.\end{aligned}$$

Exercise 17

Find the squares of the following expressions with the help of the expansion of $(a-b)^2$:

- | | | | |
|-----------|-----------|-----------|-----------|
| 1. $a-2$ | 2. $3-b$ | 3. $a-c$ | 4. $x-p$ |
| 5. $3x-y$ | 6. $4p-q$ | 7. $m-3n$ | 8. $c-4d$ |

9. $5a-2b$ 10. $5x-3y$ 11. $8b-5d$ 12. $10m-5n$
 13. $ab-c$ 14. $m-np$ 15. $xy-3x$ 16. $ab-cd$
 17. $lm-np$ 18. $2xy-a$ 19. $a-2bc$ 20. $abc-d$
 21. $l-mnp$ 22. $3a-bcd$ 23. $a+b-c$ 24. $x-y-z$
 25. $m+2n-3p$ 26. $a+b+c-d$ 27. $(l+m)-(n-p)$
 28. $(a-2b)-(3c-4d)$.

Find the squares of the following expressions with the help of the expansion of $(a+b)^2$:

29. $a-5b$ 30. $5x-2z$ 31. $2m-3n$
 32. $a+b-c$ 33. $2x+y-3z$ 34. $3p+2q-r$

Find the values:

35. 992 36. 99992 37. 4982 38. 6902 .

We can have some corollaries from the above formulae.

Corollary 1. $a^2+b^2 = a^2+b^2-2ab+2ab$
 $a^2+b^2 = (a-b)^2+2ab$

Corollary 2. $(a+b)^2 = a^2+2ab+b^2$
 $= a^2-2ab+b^2+4ab$
 $(a+b)^2 = (a-b)^2+4ab$

Corollary 3. $(a-b)^2 = a^2-2ab+b^2$
 $= a^2+2ab+b^2-4ab$
 $(a-b)^2 = (a+b)^2-4ab$

Corollary 4. $(a+b)^2+(a-b)^2 = a^2+2ab+b^2$
 $+ a^2-2ab+b^2$
 $= 2a^2+2b^2$
 $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$

Corollary 5. $(a+b)^2-(a-b)^2 = a^2+2ab+b^2$
 $-(a^2-2ab+b^2)$
 $= 4ab$
 $4ab = (a+b)^2-(a-b)^2$

Example 1. Express $25a^2 - 40ab + 16b^2$ as a perfect square.

$$\begin{aligned} 25a^2 - 40ab + 16b^2 &= (5a)^2 - 2 \cdot 5a \cdot 4b + (4b)^2 \\ &= (5a - 4b)^2 \end{aligned}$$

Example 2. If $x - y = 2$ and $xy = 24$, what is the value of $x^2 + y^2$?

$$x^2 + y^2 = (x - y)^2 + 2xy = (2)^2 + 2 \cdot 24 = 4 + 48 = 52.$$

Note: You have seen that one formula of $a^2 + b^2$ is $a^2 + b^2 = (a + b)^2 - 2ab$, apply this formula when the value of $a + b$ will be given. Here, in example 2, the value of $x - y$ is given. Hence the other formula has been used.

Example 3. If $a - b = 5$ and $ab = 24$, what is the value of $(a + b)^2$?

$$\begin{aligned} (a + b)^2 &= (a - b)^2 + 4ab = (5)^2 + 4 \cdot 24 = 25 + 96 = 121 \\ \therefore (a + b)^2 &= 121. \end{aligned}$$

Example 4. If $m + n = 10$ and $m - n = 2$, find the value of $m^2 + n^2$.

$$\begin{aligned} 2(m^2 + n^2) &= (m + n)^2 + (m - n)^2 = (10)^2 + (2)^2 \\ &= 100 + 4 = 104 \\ \therefore m^2 + n^2 &= 52 \end{aligned}$$

Example 5. If $x - \frac{1}{x} = 5$, find the value of $x^2 + \frac{1}{x^2}$.

$$\begin{aligned} x^2 + \frac{1}{x^2} &= \left(x - \frac{1}{x}\right)^2 + 2x \cdot \frac{1}{x} \\ &= (5)^2 + 2 = 25 + 2 = 27. \end{aligned}$$

Example 6. Simplify:

$$(3x-5y)^2 - 2(3x-5y)(2x-5y) + (2x-5y)^2$$

Writing $3x-5y=a$, $2x-5y=b$,

Given expression $= a^2 - 2ab + b^2$

$$= (a-b)^2 = \{(3x-5y) - (2x-5y)\}^2$$

$$= (3x-5y-2x+5y)^2$$

$$= (x)^2 = x^2$$

Example 7. Find the value after expressing as a perfect square:

$$16a^2 - 16ab + 4b^2 \text{ when } a=1, b=2.$$

$$16a^2 - 16ab + 4b^2 = (4a)^2 - 2 \cdot 4a \cdot 2b + (2b)^2$$

$$= (4a-2b)^2 = (4 \cdot 1 - 2 \cdot 2)^2$$

$$= (4-4)^2 = (0)^2 = 0.$$

Exercise 18

Simplify with the help of formula:

1. $(x+y)^2 - 2(x+y)(x-y) + (x-y)^2$

2. $(3m-n)^2 - 2(3m-n)(m-n) + (m-n)^2$

3. $(3a+4b)^2 - 2(3a+4b)(a+4b) + (a+4b)^2$

4. $(3p+2q-r)^2 - 2(3p+2q-r)(p+2q-r) + (p+2q-r)^2$

5. $(3x+2y-4z)^2 - 2(3x+2y-4z)(2x+y-3z) + (2x+y-3z)^2$

Express as a perfect square:

6. $25x^2 - 10x + 1$

7. $4a^2 - 2a + \frac{1}{4}$

8. $16m^2 - 40mn + 25n^2$

9. $4x^2 + \frac{9}{16y^2} - \frac{3x}{y}$

Express as a perfect square and find the value:

10. $999 \times 999 - 2 \times 999 \times 989 + 989 \times 989$

11. $5 \cdot 37 \times 5 \cdot 37 - 2 \times 5 \cdot 37 \times 2 \cdot 37 + 2 \cdot 37 \times 2 \cdot 37$

12. $81a^2 - 18a + 1$ when $a = -1$,

13. $16x^2 - 40xy + 25y^2$ when $x=2$, $y=-3$

14. $x^2y^2 - 10xyz + 25z^2$ when $x=1$, $y=-2$, $z=3$

15. $4(m+n)^2 - 12(m+n)p + 9p^2$ when $m=-1$, $n=2$, $p=-3$

16. $49(2x-3y)^2 - 14(2x-3y)z + z^2$ when $x=0$, $y=1$, $z=2$
 17. If $x+y=10$, $xy=21$, what is the value of $(x-y)^2$?
 18. If $a+b=7$, $ab=12$, what is the value of $(a-b)^2$?
 19. If $x - \frac{1}{x} = 3$, prove that

$$(i) \quad x^2 + \frac{1}{x^2} = 11, \quad (ii) \quad \left(x + \frac{1}{x}\right)^2 = 13.$$

20. If $m+n=12$, $m-n=2$; what is the value of m^2+n^2 ?
 21. If $p - \frac{1}{p} = m$, prove that $p^2 + \frac{1}{p^2} = m^2 + 2$.
 22. If $x+y=8$, $x-y=2$; what is the value of $4xy$?
 23. If $x+y=5$, $x-y=1$; what is the value of $8xy(x^2+y^2)$?
 24. If $a+b=-1$, $a-b=9$; find the value of $8ab(a^2+b^2)$.
 25. If $x + \frac{1}{x} = a$, prove that $\left(x - \frac{1}{x}\right)^2 = a^2 - 4$.
 26. What should be added to a^2+64 to make it a perfect square?
 27. What should be subtracted from $25a^2+36b^2$ to make it a perfect square?

5.3 Product of the sum and the difference of two terms:

Suppose that the two terms are a and b . Then their sum is $a+b$ and their difference is $a-b$. Multiplying $(a+b)$ by $(a-b)$, we get

$$\begin{aligned} (a+b)(a-b) &= a(a-b) + b(a-b) \\ &= a^2 - ab + ba - b^2 \\ &= a^2 - ab + ab - b^2 \\ &= a^2 - b^2 \end{aligned}$$

By ordinary multiplication

$$\begin{array}{r} a+b \\ a-b \\ \hline a^2+ab \\ -ab-b^2 \\ \hline a^2 \quad -b^2 \end{array}$$

In the same manner, multiply $x+y$ by $x-y$, or $r+s$ by $r-s$ and see what is the product.

$$\begin{aligned} (x+y)(x-y) &= x(x-y) + y(x-y) \\ &= x^2 - xy + xy - y^2 \\ &= x^2 - y^2 \end{aligned}$$

$$\begin{aligned}
 (r+s)(r-s) &= r(r-s) + s(r-s) \\
 &= r^2 - rs + rs - s^2 \\
 &= r^2 - s^2.
 \end{aligned}$$

Each product of the sum and difference of two terms that has been shown above has a particular pattern. Every product is the difference of the squares of the two terms that have been taken. You may take some more similar examples and verify the truth of the above statement. This general rule is expressed in the form of a formula.

$$\begin{aligned}
 &(\text{1st term} + \text{2nd term})(\text{1st term} - \text{2nd term}) \\
 &= (\text{1st term})^2 - (\text{2nd term})^2
 \end{aligned}$$

Now taking 1st term as a and 2nd term as b , we can write

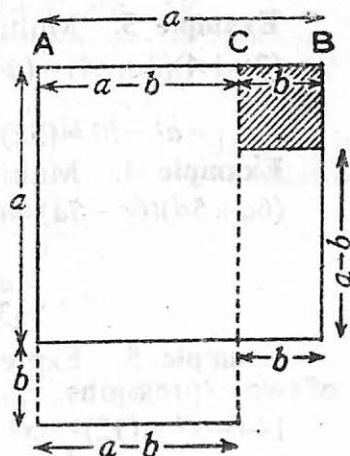
$$(a+b)(a-b) = a^2 - b^2 \quad \dots \text{ (III)}$$

As before, the truth of this formula also can be verified geometrically.

In the adjoining figure length of line segment $AB = a$.

From the square drawn on AB a square of side $BC = b$ has been removed (shaded portion).

$\therefore$ the area of the remaining portion
 $= a^2 - b^2$.



If the remaining portion is divided into two parts and brought together as shown in the figure,

we get a rectangle with length $=a+b$ and breadth $=a-b$. Hence its area $=(a+b)(a-b)$.

Thus from the geometric figure also, we get

$$(a+b)(a-b)=a^2-b^2.$$

That is the product of the sum and difference of two terms = difference of the squares of the two terms.

Note : Since $(a+b)(a-b)=a^2-b^2$, therefore $a^2-b^2=(a+b)(a-b)$. From this we find that if any expression can be expressed as the difference of the squares of two terms, that expression can be expressed as the product of two expressions. Each of these expressions is called a factor of the first expression.

Example 1. Multiply $a+5$ by $a-5$.

$$(a+5)(a-5)=a^2-5^2 \quad [\text{here it is assumed that } 5=b] \\ =a^2-25$$

Example 2. Multiply $8+b$ by $8-b$.

$$(8+b)(8-b)=8^2-b^2=64-b^2$$

Example 3. Multiply $3x+4$ by $3x-4$.

$$(3x+4)(3x-4)=(a+b)(a-b) \quad [\text{taking } 3x=a, 4=b] \\ =a^2-b^2=(3x)^2-(4)^2=9x^2-16$$

Example 4. Multiply $6c+5d$ by $6c-5d$.

$$(6c+5d)(6c-5d)=(a+b)(a-b) \\ [\text{taking } 6c=a, 5d=b] \\ =a^2-b^2=(6c)^2-(5d)^2 \\ =36c^2-25d^2.$$

Example 5. Express $144-x^2$ as the product of two expressions.

$$144-x^2=(12)^2-x^2 \\ =a^2-b^2 \quad [\text{taking } 12=a, x=b] \\ =(a+b)(a-b) \\ =(12+x)(12-x)$$

Exercise 19

Find the product with the help of formula:

1. $(a+4)(a-4)$
2. $(5+a)(5-a)$
3. $(m+8)(m-8)$
4. $(7+p)(7-p)$
5. $(2x+5)(2x-5)$
6. $(4p+1)(4p-1)$
7. $(3a+5x)(3a-5x)$
8. $(6c+7d)(6c-7d)$
9. $(m+8a)(m-8a)$
10. $(4p+b)(4p-b)$
11. $(am+n)(am-n)$
12. $(pq+r)(pq-r)$
13. $(ax+by)(ax-by)$
14. $(ab+cd)(ab-cd)$
15. $(2ab+c)(2ab-c)$
16. $(3xy+z)(3xy-z)$
17. $(3mn+4pq)(3mn-4pq)$
18. $(5pq+4rs)(5pq-4rs)$
19. $(a+b+c)(a+b-c)$
20. $(x-y+z)(x+y-z)$
21. $(2m+n+p)(2m+n-p)$
22. $(4p+2q+r)(4p+2q-r)$
23. $(a+2b+3c)(a+2b-3c)$
24. $(2x-3y+4z)(2x-3y-4z)$
25. $(3p-4q+5r)(3p-4q-5r)$

Corollary

You know $a^2 + 2ab + b^2 = (a+b)^2$

$$a^2 - 2ab + b^2 = (a-b)^2$$

Subtracting, $4ab = (a+b)^2 - (a-b)^2$ or, $ab = \frac{(a+b)^2 - (a-b)^2}{4}$ [dividing both sides by 4]

$$= \frac{(a+b)^2}{4} - \frac{(a-b)^2}{4}$$

$$= \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$$

$$\text{or, 1st term} \times 2\text{nd term} = \left(\frac{1\text{st term} + 2\text{nd term}}{2}\right)^2 - \left(\frac{1\text{st term} - 2\text{nd term}}{2}\right)^2$$

Thus, we may say that the product of two terms can be expressed as the difference of the squares of half the sum and half the difference of the two terms. That is the product of two terms can be expressed as the difference of the squares of two other terms.

Example 5. Find the product with the help of formula: 1109×1091 .

$$\begin{aligned}
 1109 \times 1091 &= (1100 + 9)(1100 - 9) \\
 &= (a + b)(a - b) \quad [\text{putting } 1100 = a, \\
 &= a^2 - b^2 \quad \quad \quad 9 = b] \\
 &= (1100)^2 - (9)^2 \\
 &= 1210000 - 81 \\
 &= 1209919
 \end{aligned}$$

Example 6. Simplify with the help of formula:

$$(x - 2y + 3z)^2 - (x + 2y - 3z)^2$$

Putting $x - 2y + 3z = a$, $x + 2y - 3z = b$,
given expression

$$\begin{aligned}
 &= a^2 - b^2 = (a + b)(a - b) \\
 &= \{(x - 2y + 3z) + (x + 2y - 3z)\} \\
 &\quad \{(x - 2y + 3z) - (x + 2y - 3z)\} \\
 &= (x - 2y + 3z + x + 2y - 3z)(x - 2y + 3z \\
 &\quad \quad \quad - x - 2y + 3z) \\
 &= (2x)(-4y + 6z) = -8xy + 12zx.
 \end{aligned}$$

Example 7. Express as product of two expressions: $49m^2 - 64n^2$

$$\begin{aligned}
 49m^2 - 64n^2 &= (7m)^2 - (8n)^2 \\
 &= (7m + 8n)(7m - 8n)
 \end{aligned}$$

Example 8. Express as the difference of the squares of two terms:

$$(3x - 2y)(x + y).$$

$$\begin{aligned}
 (3x-2y)(x+y) &= ab \quad [\text{putting } 3x-2y=a, x+y=b] \\
 &= \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 \quad [\text{from corollary}] \\
 &= \left\{ \frac{(3x-2y)+(x+y)}{2} \right\}^2 \\
 &\quad - \left\{ \frac{(3x-2y)-(x+y)}{2} \right\}^2 \\
 &= \left(\frac{3x-2y+x+y}{2} \right)^2 - \left(\frac{3x-2y-x-y}{2} \right)^2 \\
 &= \left(\frac{4x-y}{2} \right)^2 - \left(\frac{2x-3y}{2} \right)^2
 \end{aligned}$$

Exercise 20

Multiply with the help of formula:

1. 1001×999 2. 1320×1280 3. 1211×1189

Simplify with the help of formula:

4. $(x+y-z)^2 - (x-y+z)^2$ 5. $(m+n+p)^2 - (m-n-p)^2$
 6. $(2x+3y+4z)^2 - (3x-2y+4z)^2$
 7. $(a+2b-3c)^2 - (a-2b+3c)^2$
 8. $3752 \times 3752 - 3742 \times 3742$
 9. $(57632)^2 - (57627)^2$ 10. $(665 \cdot 67)^2 - (334 \cdot 33)^2$
 11. $987654 \times 987654 - 987653 \times 987653$

Express as product of two expressions:

12. $x^2 - 4y^2$ 13. $16a^2 - c^2$ 14. $16m^2 - 25n^2$
 15. $81p^2 - 100n^2$ 16. $x^2 - 1$ 17. $4 - p^2$
 18. $(a+b)^2 - (c+d)^2$ 19. $(2x+3y)^2 - (x-y)^2$
 20. $a^2 + 2ab + b^2 - 1$ 21. $x^2 - 4xy + 4y^2 - 4$

Express as difference of two squares:

22. 87×13 23. 321×209
 24. $(3x+2y)(x+4y)$ 25. $(5m-3y)(3m+2y)$

Chapter 6

Simple Factorisation

6.1 In arithmetic you have already learnt what we mean by a factor or multiple of a given number. Expressing any number as the product of two or more prime numbers we can get its factors. For example, $21 = 3 \times 7$; hence 1, 3, 7, $(3 \times 7 =) 21$, are factors of 21. Likewise, $105 = 3 \times 5 \times 7$; hence 1, 3, 5, 7, $(3 \times 5 =) 15$, $(3 \times 7 =) 21$, $(5 \times 7 =) 35$, $(3 \times 5 \times 7 =) 105$ are all factors of 105.

In the same manner we can find the factors in algebra. For example $ab = a \times b$.

Similarly, $abc = a \times b \times c$.

You know that an algebraic expression may have one or more terms. For example

- (1) a, ab, x^2y, xyz , etc. are monomials;
- (2) $a+b, a^2-ab, x^2y+xyz$, etc. are binomials;
- (3) $a^2b+ab+abc, a^3+b^3+c^3$, etc. are polynomials.

In case of monomials, it is easy to find the factors by inspection. In case of binomials or polynomials the factors are formed with the help of fundamental ideas like commutative, associative, etc. laws and the formulae that you have learnt in the last chapter.

For example $ab+bc = ab+cb = (a+c)b$.

Algebraic expressions may be monomials or

polynomials and so the factors may also be monomials or polynomials. But since the arithmetical numbers are all monomials their factors are always monomials.

In finding factors of algebraic expressions we will use commutative and associative laws of multiplication. These laws have been applied in the following examples.

Example 1. Factorise: $6+2a$
 $6+2a=2.3+2a=2(3+a)$ [by associative law of multiplication]

Example 2. Factorise: $xy-yz$.
 $xy-yz=xy-zy$ [by commutative law of multiplication]
 $= (x-z)y$.

Example 3. Factorise: $4ab^2+6a^2b$
 $4ab^2+6a^2b=2.2ab.b+2.3a.ab$
 $=2.b.2ab+3a.2ab$
 $=(2b+3a).2ab$
 $=2ab(2b+3a)$.

Example 4. Factorise: $a^2bc+ab^2c+abc^2$
 $a^2bc+ab^2c+abc^2=a(abc)+b(abc)+c(abc)$
 $=(a+b+c)abc$.

Exercise 21

Factorise:

- | | | |
|-------------------------------------|-----------------|----------------------------|
| 1. $8+4x$ | 2. $3a+6$ | 3. $2a+4b$ |
| 4. $5p-10q$ | 5. $ab+ac$ | 6. $xy-zx$ |
| 7. $2mn+4mp$ | 8. $axy-bxy$ | 9. $abc-bcd$ |
| 10. x^2y+xy^2 | 11. a^2b-a^2c | 12. $x^2yz+xy^2z+xyz^2$ |
| 13. $amn-bmn+cmn$ | | 14. $p^2a^2+p^2b^2+p^2c^2$ |
| 15. $a^2b^2m^2+a^2b^2n^2+a^2b^2p^2$ | | |

6.2 Factorisation with the help of formula :

You have seen in the previous chapters that $a^2 - b^2 = (a+b)(a-b)$; that is an algebraic expression of the form $a^2 - b^2$ can be expressed as the product of $(a+b)$ and $(a-b)$. Hence each of the expressions $(a+b)$ and $(a-b)$ is a factor of $a^2 - b^2$. If any algebraic expression is in the form $a^2 - b^2$, it can be factorised with the help of the above formula.

Followings are some examples of factorisation with the help of formula.

Example 1. Factorise: $a^2 - 9$.

$$\begin{aligned} a^2 - 9 &= a^2 - 3^2 \\ &= a^2 - b^2 \quad [\text{putting } 3 = b] \\ &= (a+3)(a-3) \quad [\text{putting the value of } b]. \end{aligned}$$

Example 2. Factorise: $25 - x^2$.

$$\begin{aligned} 25 - x^2 &= 5^2 - x^2 \\ \text{Putting } a &= 5, x = b, \text{ we get} \\ 25 - x^2 &= a^2 - b^2 = (a+b)(a-b) \\ &= (5+x)(5-x) \quad [\text{putting values of } a \text{ and } b]. \end{aligned}$$

Example 3. Factorise: $25x^2 - 16$.

$$\begin{aligned} 25x^2 - 16 &= (5x)^2 - (4)^2 \quad [\text{putting } 5x = a, 4 = b] \\ &= a^2 - b^2 \\ &= (a+b)(a-b) \\ &= (5x+4)(5x-4) \quad [\text{putting values of } a \\ &\quad \text{and } b]. \end{aligned}$$

Example 4. Factorise: $64m^2 - 49n^2$

$$\begin{aligned} 64m^2 - 49n^2 &= (8m)^2 - (7n)^2 \\ &= a^2 - b^2 \quad [\text{putting } 8m = a, 7n = b] \\ &= (a+b)(a-b) \\ &= (8m+7n)(8m-7n) \end{aligned}$$

Example 5. Factorise:

$$25(3a-2b)^2 - 16(2a+3b)^2.$$

$$\begin{aligned} 25(3a-2b)^2 - 16(2a+3b)^2 &= 5^2(3a-2b)^2 - 4^2(2a+3b)^2 \\ &= \{5(3a-2b)\}^2 - \{4(2a+3b)\}^2 \\ &= (15a-10b)^2 - (8a+12b)^2 \end{aligned}$$

If $15a-10b=x$, $8a+12b=y$,

given expression

$$\begin{aligned} &= x^2 - y^2 = (x+y)(x-y) \\ &= \{(15a-10b) + (8a+12b)\} \\ &\quad \{(15a-10b) - (8a+12b)\} \\ &= (23a+2b)(7a-22b) \end{aligned}$$

Example 6. Factorise:

$$100am^2 - 49a(2m-3n)^2$$

$$\begin{aligned} &100am^2 - 49a(2m-3n)^2 \\ &= a\{100m^2 - 49(2m-3n)^2\} \\ &= a[(10m)^2 - \{7(2m-3n)\}^2] \\ &= a[(10m)^2 - (14m-21n)^2] \\ &\quad [\text{putting } 10m=x, \quad 14m-21n=y] \\ &= a(x^2 - y^2) = a(x+y)(x-y) \\ &= a\{10m + (14m-21n)\}\{10m - (14m-21n)\} \\ &= a(10m+14m-21n)(10m-14m+21n) \\ &= a(24m-21n)(21n-4m) = 3a(8m-7n)(21n-4m) \end{aligned}$$

Example 7. Factorise: $a^2 - 2ab - c^2 + 2bc$.

Note that this expression can not be expressed in the form $a^2 - b^2$ straight away. Now we have to see whether it can be expressed in the form $a^2 - b^2$ by adding and subtracting any term. First part

of the expression is $a^2 - 2ab$. Adding b^2 with it we get $a^2 - 2ab + b^2$. If we add b^2 with the expression we will have to subtract b^2 from it so that its value remains unchanged. Now let us see what do we get.

$$\begin{aligned} a^2 - 2ab - c^2 + 2bc &= a^2 - 2ab + b^2 - b^2 - c^2 + 2bc \\ &= (a^2 - 2ab + b^2) - (b^2 + c^2 - 2bc) \end{aligned}$$

[by rule of bracket]

$$= (a-b)^2 - (b-c)^2,$$

[which is obviously of the form $a^2 - b^2$]

$$= \{(a-b) + (b-c)\} \{(a-b) - (b-c)\}$$

$$= (a-b+b-c)(a-b-b+c)$$

$$= (a-c)(a-2b+c)$$

Exercise 22

Factorise with the help of formula:

- | | | |
|-------------------------|-----------------------------|--------------------|
| 1. $a^2 - 16$ | 2. $4 - m^2$ | 3. $p^2 - 49$ |
| 4. $81 - c^2$ | 5. $25m^2 - 16$ | 6. $100 - 36d^2$ |
| 7. $1 - 9c^2$ | 8. $4a^2 - 9b^2$ | 9. $16p^2 - 49q^2$ |
| 10. $100x^2 - 121y^2$ | 11. $a^2b^2 - c^2$ | |
| 12. $m^2n^2 - 4$ | 13. $25 - a^2b^2c^2$ | |
| 14. $m^2n^2 - p^2q^2$ | 15. $9p^2q^2 - 16r^2s^2$ | |
| 16. $1 - 9a^2b^2c^2d^2$ | 17. $12a^2 - 27x^2$ | |
| 18. $80x^2 - 125y^2$ | 19. $ax^2 - ay^2$ | |
| 20. $9m^2 - 81n^2$ | 21. $m^2p^2 - 4m^2q^2$ | |
| 22. $(x+y)^2 - 4$ | 23. $1 - (m+n)^2$ | |
| 24. $(a+b)^2 - (a-b)^2$ | 25. $100 - (2a+3b)^2$ | |
| 26. $(3x+4y)^2 - 64x^2$ | 27. $(4m+3n)^2 - (3m-2n)^2$ | |

28. $9(5c+4d)^2-4(3c-2d)^2$
 29. $64ax^2-49a(x-2y)^2$
 30. $(2a-b-c)^2-(a+2b-c)^2$
 31. $(3x-4y+7z)^2-(2x-3y+5z)^2$
 32. $(x^2+xy+y^2)^2-(x^2-xy+y^2)^2$
 33. $9(3m+2n-p)^2-4(2m-3n+4p)^2$
 34. $(m+n-p+q)^2-(m-n+p-q)^2$
 35. $(2a+3b-4c+5d)^2-(2a-3b+4c-5d)^2$
 36. $4b^2c^2-(b^2+c^2-a)^2$
 37. $a^2-b^2+c^2-d^2-2(ac+bd)$
 38. $x^2(y-z)^2-y^2(z-a)^2$
 39. $x^2y^2(a+b)^2-p^2q^2(a-b)^2$
 40. $a^2+b^2+2ab-c^2$
 41. $a^2-b^2+2bc-c^2$
 42. $x^2-y^2+4yz-4z^2$
 43. $9m^2-n^2-p^2+2np$
 44. $2a^2-7a^2-b^2+c^2$
 45. $16x^2-4a^2-12ab-9b^2$
 46. $a^2-1+2b-b^2$
 47. a^2-2a-b^2+2b
 48. $a^2-b^2-2ac+2bc$
 49. $(a^2-b^2)(c^2-d^2)-4a^2bcd$
 50. $(a^2-b^2-c^2+d^2)^2-4(ad-bc)^2$
-

Chapter 7

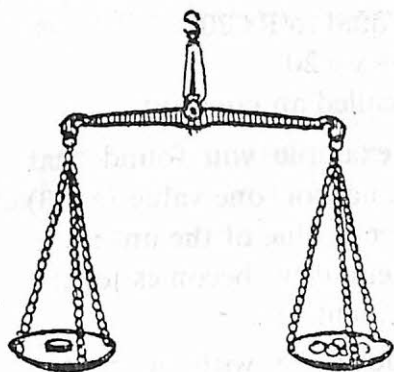
Forming Simple Equation and its solution

7.1 Forming equation:

Suppose that you have 10 marbles and your brother has 8 marbles. That means the number of your marbles and that of your brother is not same. If the number of marbles of both of you is to be made equal, your brother should have 2 more marbles. That is at that time $10 = 8 + 2$. Or you should have 2 marbles less. That is at that time $10 - 2 = 8$. Note that 2 has been added to less marbles and same 2 has been subtracted from more marbles. In both the cases both sides have been made equal. Now if you are asked, 'how many more marbles should your brother have to make the number equal to the number of your marbles' or 'how many marbles should be removed from your marbles so that both of you may have same number of marbles', in both the cases that unknown number of marbles will be 2.

Again, note the following example. Suppose that you will buy 500gms of potato from the market. The shopkeeper will put 500gms weight on one pan and will go on putting potato on the other till the scale is horizontal. For being horizontal the quantity of potato has to be equal to the weight of 500gms-weight. This type of making both sides 'equal' is called equation in common terms. If

any unknown term or expression is used to make both sides equal and if the equality holds for a definite value of the unknown term, then the equality is called an equation and the determination of the



value of the unknown term is called solution of equation. Now let us see how an equation can be formed.

Suppose, your mother sends you to the market with Rs 20. From that you buy rice for Rs10, fish for Rs 4,

potato for Rs 2 and other things for Re 1. Hence you have purchased articles for Rs $(10+4+2+1)$ or Rs 17. This sum of Rs 17 is not equal to Rs 20. How many rupees are to be added to Rs 17 to make the sum equal to Rs 20? That is definitely Rs 3. You are left with Rs 3. Hence, the sum of the money that you have spent on shopping and that left with you is the total amount of money. That is, $20 = 17 + 3$. In the above example you went to the market with Rs 20. From that after buying certain articles of different kinds you are left with some amount of money. You can find out the total amount of money spent on shopping by arithmetical methods. But suppose that you have lost the money left with you on way home. That is, you donot know the amount of money that should have been with you. In algebra you have learnt to express this type of indefinite

number by x or any other symbol. Let us suppose that the money left with you be Rs x . Hence, the expenditure of Rs 17 and remaining Rs x will together be Rs $(17+x)$. Since you went to the market with Rs 20, Rs $(17+x)$ must be equal to Rs 20.

$$\therefore 17+x=20$$

The way it is written is called an equation.

Note : In the above example you found that both sides have become equal for one value ($x=3$) of x . In certain cases for any value of the unknown term both sides of the equality becomes equal. Those equalities are called identities.

Example 1. If you add Rs 5 with the money that you have, you will have altogether Rs 25. Express this statement by an equation.

Suppose, you have Rs x . If Rs 5 are added to it the total amount of money will be Rs $(x+5)$. Since this total is equal to Rs 25, we can write

$$x+5=25$$

This is the required equation.

Example 2. On your asking the teacher about marks scored in mathematics, he replied, 'if you would have scored 10 marks more than what you have scored, your total marks would have been 85'. Form the equation.

Suppose, you have scored x marks. If you would have scored 10 marks more, your marks would have been $x+10$. But your marks would have been 85 if you would have scored 10 marks more. Then $x+10$ will be equal to 85. That is,

$$x+10=85.$$

This is the required equation.

Example 3. Adding 8 with 5 times a number it becomes 38. Find the equation.

Let the number be x . Its 5 times is $5x$, adding 8 with it we get $5x+8$. Hence from the given condition, $5x+8=38$.

This is the required equation.

Example 4. Bachchu, Bubai and Baban have altogether 56 marbles. Number of Bubai's marbles is half of that of Bachchu and number of Baban's marbles is half that of Bubai. Form the equation necessary to find the number of Bachchu's marbles.

Suppose, Bachchu had x marbles. Number of Bubai's marbles is half that of Bachchu. Hence number of Bubai's marbles $= \frac{x}{2}$.

Again, number of Baban's marbles is half that of Bubai. Hence number of Baban's marbles

$$= \frac{1}{2} \times \frac{x}{2} = \frac{x}{4}.$$

The sum of marbles of the three is 56. Hence

$$x + \frac{x}{2} + \frac{x}{4} = 56; \text{ this is the required equation.}$$

Example 5. At present you are 8 years older than your younger brother. 4 years hence your age will be double that of your brother. Form an equation to find the ages of both of you.

Let the present age of your younger brother be x years. Since your age is 8 years more than that of your younger brother, your present age is $(x+8)$ years. 4 years hence the age of each of you will be 4 years more.

Thus at that time your age will be $(x+8)+4 = (x+12)$ years and your brother's age will be $(x+4)$ years.

Your age will be twice your brother's age. Hence, your age after 4 years can be obtained by doubling your brother's age after 4 years.

That is, $2(x+4) = x+12$; this is the required equation.

Exercise 23

Form equations to solve the following problems:

1. If 45 is subtracted from a number, the difference is 100. What is the number?
2. If 67 is added to a number the sum is 200. What is the number?
3. If a number is multiplied by 15 the product is 375. What is the number?
4. If a number is divided by 12, the quotient is 432. What is the number?
5. If the sum of one-third and one-fourth of a number is 35, what is the number?
6. If 20 is added to 5 times a number the sum is 100. What is the number?
7. If 63 is subtracted from 11 times a number the difference is 190. What is the number?
8. If 15 is added to 5 times a number the sum is 6 times the number. What is the number?
9. In a certain examination Manoara scored 7 marks more than Indrani in English and the sum of their total marks is 117. What was the mark scored by each?
10. In a certain cricket match of the school John made 8 runs more and Maidul made 5 runs less than that of Baren. The three made altogether 108 runs. What were the runs of each?

11. Sumanta went to market with some money. He bought rice worth $\frac{1}{2}$ of the money and vegetables worth $\frac{1}{4}$ th of the money and was left with Rs 3. With what amount of money did he go to the market?

12. Kamal, Bansidhar and Haripada work in a workshop. Daily wage of Bansidhar is Rs 2 less than that of Kamal and daily wage of Haripada is Rs 3 less than that of Bansidhar. Their total wage is Rs 38. What is the daily wage of each?

13. Total marks scored by Ayesa, Nasifa and Sayrah in Mathematics in the examination of class VII was 156. Nasifa scored 9 marks more than Ayesa and Sayrah scored 9 marks less than Ayesa. What were the marks scored by each?

14. After spending $\frac{1}{3}$ of his pay Mr. E. S. Friskin found that he was left with Rs 1100. What is his pay?

15. Sum of the ages of father and son is 42 years at present. Age of father is 5 times the age of son. What is the age of each?

16. From the market you purchased 2 kg potato at Rs 2 per kg., 2kg. 'ata' at Rs 3 per kg. and some rice at Rs 3 per kg. You spent altogether Rs 28. What weight of rice did you purchase?

17. Divide 70 marbles among Tutun, Tukaj and Tumpa in such a way that Tukaj gets half that of Tutun and Tumpa gets half that of Tukaj. What is the share of each?

18. Of the total number of fruits with a fruit vendor $\frac{1}{4}$ was apple, $\frac{1}{8}$ was banana, $\frac{1}{4}$ was orange and the rest 165 was lichi. What was the total number of fruits with the vendor?

19. A school spent Rs 2095 on an educational excursion. There were 3 teachers and 60 students in the group. Expenditure on each teacher was Rs 40, on each student of higher class was Rs 35 and on each student of lower class was Rs 30. What was the number of students of higher and lower classes?

7.2 Solution of equation:

You have learnt how to form an equation. Now we will consider how to find the value of the unknown term involved in that equation; that is

we will learn the method of solving an equation. We generally take help of some axioms for solving an equation.

Axiom 1. If equal quantities are added to equal quantities, the sums will also be equal.

Axiom 2. If equal quantities are subtracted from equal quantities, the differences will also be equal.

Axiom 3. If equal quantities are multiplied by equal quantities, the products will also be equal.

Axiom 4. If equal quantities are divided by equal quantities, the quotients will also be equal.

Apart from these axioms some rules are to be followed in finding the value of unknown term.

(i) The unknown term is generally kept in the left hand side of the equation.

(ii) If any term is brought from the left-hand side to the right-hand side or vice versa the sign of the term has to be changed. This is called transposition. Actually, this is a different form of axioms 1 and 2

Note : You can easily verify the truth of these statements:

Variable quantity: Number of students present everyday in your class or in any other class of your school changes from day to day. Generally, the total number of students present on a particular day is different from that present on another day. That is, the number of students present is a changing quantity. Take another example. Sup-

pose, you save 10 paise everyday from your tiffin money. Then your savings will be 10 paise on the first day, 20 paise on the second day, 30 paise on the third day and so on. Thus the amount of your savings is also a changing quantity. This changing quantity, whose value is not fixed, is a variable quantity.

Example 1. Solve : $5x - 2 = 8$

$$5x - 2 = 8$$

Adding 2 to both sides, we get

$$5x - 2 + 2 = 8 + 2$$

$$\text{or, } 5x = 10$$

Dividing both sides by 5, we get

$$\frac{5x}{5} = \frac{10}{5}$$

$$\text{or, } x = 2$$

$\therefore$ required solution is $x = 2$

Note : Instead of adding 2 to both sides if 2 is brought from left-hand side to the right-hand side by transposition, -2 will become $+2$ in the right-hand side. That is, we will have $5x = 8 + 2$.

Example 2. Solve: $8x - 3 = 3x + 17$.

$$8x - 3 = 3x + 17$$

By transposition, we get

$$8x - 3x = 17 + 3$$

$$\text{or, } 5x = 20$$

$$\text{or, } \frac{5x}{5} = \frac{20}{5} \text{ or, } x = 4.$$

[Terms with x are kept in the left-hand side and those without x are kept in the right-hand side.]

Example 3. Solve: $3(6-2x)+4(5x-1)=0$
 $3(6-2x)+4(5x-1)=0$

or, $18-6x+20x-4=0$

or, $14x+14=0$ or, $14x=-14$

[by transposition]

or, $x = \frac{-14}{14}$ or, $x = -1$

Example 4. Solve : $\frac{2x}{3} - \frac{x}{4} = \frac{x}{5} - \frac{x}{6} + 23$

$\frac{2x}{3} - \frac{x}{4} = \frac{x}{5} - \frac{x}{6} + 23$

or, $\frac{2x}{3} - \frac{x}{4} - \frac{x}{5} + \frac{x}{6} = 23$ [by transposition]

or, $\frac{40x-15x-12x+10x}{60} = 23$ or, $\frac{23x}{60} = 23$

or, $60 \times \frac{23x}{60} = 60 \times 23$ [multiplying both sides by 60]

or, $23x = 60 \times 23$ or, $x = \frac{60 \times 23}{23}$

or, $x = 60$

Exercise 24

Solve the following equations:

1. $8x=40$

2. $9x=-81$

3. $-7x=-35$

4. $\frac{x}{4}=-4$

5. $6x+3=15$

6. $3x-7=-22$

7. $9x+8=7x+28$

8. $15x-9=11x+35$

9. $3(2x-1)+7x=-29$

10. $5(x-3)+4(2x-6)=0$

11. $13(2x+5)=11(3x+4)$

12. $8\left(\frac{x}{2}-3\right)=8$

13. $3(2x+5)-2x=3(x+6)$ 14. $4(3x+1)=15x-7$
15. $5x+2(2x-1)=3x-8$ 16. $14(x-3)+3(x+4)=4$
17. $8(2x-7)-9(3x-14)=15$
18. $50+3(4x-5)+8(x+3)=x+2$
19. $\frac{x}{2}+\frac{1}{3}=\frac{x}{3}+\frac{1}{2}$ 20. $\frac{x}{2}-2=\frac{x}{4}+\frac{x}{5}-1$
21. $\frac{3x+1}{16}+\frac{2x-3}{7}=\frac{x+3}{8}+\frac{3x-1}{14}$
22. $ax+b=cx+d$ 23. $\frac{x}{a}+a=\frac{x}{b}+b$
24. $\frac{ax}{b}-\frac{bx}{a}=a^2-b^2$
25. $\frac{1}{2}(x+1)+\frac{1}{3}(x+2)+\frac{1}{4}(x+3)=16$
26. $\frac{4x+3}{9}+\frac{13x}{108}=\frac{8x+19}{18}$
27. Solve the equations formed in the previous exercise.
-

Answers

Exercise 1

1. (i) $x+y$ (ii) $a-b$ (iii) $10-k$ (iv) $5+r$ (v) $15x$
 (vi) $2x-5$ (vii) $\frac{b}{7}$ (viii) $x+y-10$ (ix) $z < 10$ (x) $x=y$
2. (i) sum of p and q (ii) difference between z and y
 (iii) 5 times a (iv) one-sixth of x (v) sum of x , y and z
 (vi) sum of two times p and r (vii) sum of m and double of n
 (viii) difference between a and three times b (ix) sum of one-third of p and 6 (x) one-third of the sum of a and two times b .
3. $5+x$ 4. $15-x$ 5. (i) $(60-x)$ years (ii) $(60+y)$ years
6. $(x+3)$ years 7. Rs $4x$ 8. $x-10$ 9. $18-y$ 10. $\frac{64}{p}$
11. $(x+y)$ km. 12. $2x$, Rs $\frac{2x}{5}$ 13. $\frac{200-x}{y}$ 14. Rs $15x$, $\frac{25}{x}$ kg.
15. (i) $1000a$ metres, $\frac{y}{1000}$ km. 16. $\frac{x}{60}$ hrs. 17. $(x-5)$ years, $(x+15)$ yrs. 18. $10x$ 19. Rs $50x$, Rs $\frac{50x}{y}$ 20. $10y+x$
21. Rinku's present age is $(y-x)$ years ; Mitu's age is $(y+5)$ yrs, Rinku's age is $(y-x+5)$ yrs.

Exercise 2

1. 16 2. 11 3. 10 4. $2\frac{1}{2}$ 5. 15 6. 3 7. 35
8. $2\frac{1}{2}$ 9. $2\frac{3}{8}$ 10. 1200 11. 39 12. 88 13. 84
14. 9 15. $5\frac{9}{10}$ 16. 1 17. $1\frac{1}{3}$ 18. $6\frac{1}{4}$ 19. $29\frac{1}{2}$
20. 11 21. 56.

Exercise 3

1. (i) Profit of Rs 10 (ii) decrease of weight by 20 kg. (iii) rise of temperature by 5° (iv) above 3 m. (v) 12 km. to the west
 (vi) Kamal earns Rs 5 less than Karim
2. (i) an expenditure of Rs 5 (ii) a capital of Rs 12 (iii) 3 km to the south (iv) increase of weight by 2 kg (v) 5 km to the east
 (vi) a fall of temperature by 3° . 4. (i) -10, -2, 0, 5 (ii) -16, -13, -12, 0 (iii) -10, -8, 5, 7 5. (i) 5 (ii) 10 (iii) 100
 (iv) 15 (v) 200 6. 23rd December 7. Rs 55.

Exercise 4

2. (i) 3 (ii) -8 (iii) -2 (iv) -9 (v) 7
 3. (i) 4 (ii) -8 (iii) 0 (iv) -1 4. (i) 0 (ii) 0
 (iii) -6 (iv) -17.

Exercise 5

1. (i) 9 (ii) -18 (iii) -5 (iv) 9 (v) -40 (vi) 25
 (vii) -15 (viii) -20 (ix) 0 (x) -16
 2. (i) -5 (ii) -16 (iii) -4 (iv) -57 (v) -100.

Exercise 6

1. -12 2. -6 3. -10 4. 96 5. 20 6. 15
 7. -30 8. 24 9. 100 10. 30.

Exercise 7

1. x^5 2. x^6 3. a^{15} 4. a^{13} 5. a^3 6. $-a^4$ 7. 64 8. 243
 9. 3125 10. -1600 11. (i) 14 (ii) -18 12. (i) 4 (ii) 16
 (iii) 17 13. (i) -2 (ii) 4 (iii) -4.

Exercise 8

1. -4 2. -7 3. -5 4. -10 5. 8 6. -9 7. 5 8. 11.

Exercise 9

3. (i) a^2+6a (ii) $12a^2+18a$ (iii) $a^3b+a^2b^2$
 (iv) $a^2bc+ab^2c+abc^2$ 4. (i) 3 (ii) -1 (iii) 1 (iv) 11.

Exercise 10

1. (i) $5a$ (ii) $12x$ (iii) $3abc$ (iv) $4a^2b$ (v) $12x^2$ (vi) $7x$
 (vii) $-10x$ (viii) $6x^2+y^2$ (ix) $2x^2+3y^2$ (x) 0 2. (i) $2a+4b+c$
 (ii) $5x+10y+4z$ (iii) $7y-3z$ (iv) $15x^2+5$ (v) $12m^2-10mn+6p^2$
 (vi) 0 (vii) $16x^3+7xy+16y^2$ (viii) $-abc^2+a^2bc$ 3. (i) $9a+3b$
 (ii) $b+c$ (iii) $4mn$ (iv) $2b^2$.

Exercise 11

1. (i) $-a$ (ii) $-7a$ (iii) $6a^2$ (iv) $-abc$ (v) $4ab^2$
 2. (i) $3a-2b$ (ii) $2a^2-2b^2$ (iii) $5x^2+6x$ (iv) $3x^2+7x-5$
 (v) $-ab^2+5ab+6$ 3. $-8x-5$ 4. $-n^2$.

Exercise 12

1. $6a^3$ 2. $-9x^5$ 3. $24m^7$ 4. $-6a^3b^4$ 5. $-15ab$
6. $-56a^8b^3c$ 7. $-27a^8b^8c^8$ 8. $30a^4b^5c^8$ 9. $30p^4q^8r^5$
10. $75m^{11}n^8t^8$ 11. $-30abc$ 12. $-120a^5b^3$ 13. $6a^4b^6$ 14. $-\frac{1}{8}a^{11}$
15. $30a^2b^2c^2$ 16. $-24a^3b^3c^3$ 17. $a^3b^3c^3$ 18. $24a^5b^7$
19. $-54x^6y^7z^6$ 20. $12m^7n^9$ 21. $-30abcx^2yz$ 22. $-8a^4b^3c^4$
23. $-12a^2b^2c^2d^3$ 24. $-a^6b^2c^4$ 25. $-48p^5q^4r^8s^2t^2$

Exercise 13

1. $3x^2+7x+2$ 2. $12x^2-2x-24$ 3. $x^3+x^2y+xy^2+y^3$
4. $2x^3+2ax^2-ax-a^2$ 5. $10a^5+15a^4b+12ab+18b^2$
6. $-8a^3-8a^2b+6a^2+6ab$ 7. $9b-5ab-27a^2+15a^3$
8. $99a^4+12a^3-15a^2$ 9. $3mx^3+3nx^2y-7mxy^2-7ny^3$
10. $-22a^4+14a^3+33a^2-21a$ 11. $63x^7+77x^6+72x^5+88x^4$
12. $x^2y-xy^2-xyz+y^2z$ 13. $a^3x^3+ab^2x^3+a^2by^3+b^3y^3$
14. $16x^6-114x^3y^2+143y^4$ 15. $-abx^4+adx^2-bcx^2+cd$
16. (i) 0 (ii) 0 (iii) $a^2b-ab^2+b^2c-bc^2+c^2a-ca^2$
17. $l^2x+m^2y+n^2z+lmx+mny+nlz+l^2+m^2+n^2+lm+mn+nl$
18. $2a^2x+2b^2y+2c^2z+abx+acx+aby+bcy+acz+bcz$

Exercise 14

1. $2b-a$ 2. $4a^2+5a+3$ 3. $3b-2ab^2-a^2b$ 4. $-2a^3x^3$
- $+a^2x^2-ax$ 5. $-2m^3p^2+2m^2p^3-2mp^3$ 6. $3p^3q^2-9p^2q+6p$
7. $-\frac{1}{4}a^2b^2c^2$ 8. $\frac{5}{2}ab^3-\frac{2}{3}b^2c^3$ 9. $\frac{3}{4}b-\frac{5}{4}a+3ab$ 10. $-\frac{1}{3}n^2-\frac{1}{8}m$
- $+\frac{1}{3}n-\frac{1}{3}m^2$ 11. $-1-\frac{8}{3}pr+3p^2qr$

Exercise 15

1. a^2+6a+9 2. $64+16b+b^2$ 3. $1+2c+c^2$
4. $a^2+2ax+x^2$ 5. $m^2+2mp+p^2$ 6. $36x^2+12x+1$
7. $9a^2+30a+25$ 8. $1+18c+81c^2$ 9. $m^2+4mn+4n^2$
10. $a^2+6ab+9b^2$ 11. $p^2+8pq+16q^2$ 12. $16x^2+40xy+25y^2$
13. $64a^2+48ab+9b^2$ 14. $25c^2+70cd+49d^2$ 15. $100m^2$
- $+100mn+25n^2$ 16. $a^2b^2+2abc+c^2$ 17. $p^2+2pqr+q^2r^2$
18. $x^2y^2+4xyz+4z^2$ 19. $9a^2+6abc+b^2c^2$ 20. $l^2m^2+2lmnp$
- $+n^2p^2$ 21. $4x^2y^2+4xyz+z^2$ 22. $a^2+6abc+9b^2c^2$ 23. $a^2b^2c^2$
- $+2abcd+d^2$ 24. $9x^2+6mnp+x+m^2n^2p^2$ 25. $a^2+2ab+b^2$
- $+16a+16b+64$ 26. $m^2+n^2+p^2+2mn+2np+2pm$ 27. a^2+x^2

- $+4p^2+2ax+4xp+4pa$ 28. $x^2+4y^2+z^2+4xy+4yz+2zx$
 29. $9m^2+4n^2+p^2+12mn+4np+6pm$ 30. $a^2+b^2+c^2d^2+2ab$
 $+2acd+2bcd$ 31. $x^2+4y^2+z^2p^2+4xy+2xzp+4yzp$
 32. $p^2+q^2+r^2+2pq+2qr+2rp+10p+10q+10r+25$
 33. $l^2+4m^2+n^2+9p^2+4lm+2ln+6lp+4mn+12mp+6np$
 34. $1+4a^2+9b^2+16c^2+4a+6b+8c+12ab+16ac+24bc$
 35. $a^2+4b^2+9c^2+d^2+4ab+6ac+2ad+12bc+4bd+6cd$
 36. $m^2+n^2+4p^2+9q^2+2mn+4mp+6mq+4np+6np+12pq$

Exercise 16

1. $4m^2$ 2. $36x^2$ 3. $16a^2$ 4. $16a^2$ 5. $x^2+2xy+y^2$
 6. $(3a+1)^2$ 7. $(2a+\frac{1}{2})^2$ 8. $(2a+3b)^2$ 9. $(1+\frac{2}{a})^2$
 10. $(5a+4b)^2$ 11. $(131+69)^2$, 40000 12. $(6.67+3.33)^2$, 100
 13. $(7a+1)^2$, 64 14. $(3x+4y)^2$, 1 15. $(6m+11n)^2$, 49
 16. $(8a-1)^2$, 0 17. 7 18. 34 19. 104 20. 50
 23. 23 24. 47.

Exercise 17

1. a^2-4a+4 2. $9-6b+b^2$ 3. $a^2-2ac+c^2$ 4. x^2
 $-2px+p^2$ 5. $9x^2-6xy+y^2$ 6. $16p^2-8pq+q^2$ 7. m^2-6mn
 $+9n^2$ 8. $c^2-8cd+16d^2$ 9. $25a^2-20ab+4b^2$ 10. $25x^2$
 $-30xy+9y^2$ 11. $64b^2-80bd+25d^2$ 12. $100m^2-100mn$
 $+25n^2$ 13. $a^2b^2-2abc+c^2$ 14. $m^2-2mnp+n^2p^2$ 15. x^2y^2
 $-6x^2y+9x^2$ 16. $a^2b^2-2abcd+c^2d^2$ 17. $l^2m^2-2lmnp$
 $+n^2p^2$ 18. $4x^2y^2-4axy+a^2$ 19. $a^2-4abc+4b^2c^2$ 20. $a^2b^2c^2$
 $-2abcd+d^2$ 21. $l^2-2lmnp+m^2n^2p^2$ 22. $9a^2-6abcd+b^2c^2d^2$
 23. $a^2+b^2+c^2+2ab-2ac-2bc$ 24. $x^2+y^2+z^2-2xy-2xz$
 $+2yz$ 25. $m^2+4n^2+9p^2+4mn-6mp-12np$ 26. $a^2+b^2+c^2$
 $+d^2+2ab+2ac-2ad+2bc-2bd-2cd$ 27. $l^2+m^2+n^2+p^2$
 $+2lm-2ln+2lp+2mp-2mn-2np$ 28. $a^2+4b^2+9c^2+16d^2$
 $-4ab-6ac+8ad+12bc-16bd-24cd$ 29. $a^2-10ab+25b^2$
 30. $25x^2-20xz+4z^2$ 31. $4m^2-12mn+9n^2$ 32. $a^2+b^2+c^2$
 $+2ab-2ac-2bc$ 33. $4x^2+y^2+9z^2+4xy-12xz-6yz$ 34. $9p^2$
 $+4q^2+r^2+12pq-6pr-4qr$ 35. 9801 36. 99980001 37. 248004
 38. 476100.

Exercise 18

1. $4y^2$ 2. $4m^2$ 3. $4a^2$ 4. $4p^2$ 5. $x^2+y^2+z^2+2xy$
 $-2xz-2yz$ 6. $(5x-1)^2$ 7. $(2a-\frac{1}{2})^2$ 8. $(4m-5n)^2$
 9. $(2x-\frac{3}{4y})^2$ 10. $(999-989)^2, 100$ 11. $(5\cdot37-2\cdot37)^2, 9$
 12. $(9a-1)^2, 100$ 13. $(4x-5y)^2, 529$ 14. $(xy-5z)^2, 289$
 15. $(2m+2n-3p)^2, 121$ 16. $(14x-21y-z)^2, 529$ 17. 16
 18. 1 20. 74 22. 60 23. 624 24. -6560 25. $16a$
 or $-16a$ 27. $60ab$ or $-60ab$.

Exercise 19

1. a^2-16 2. $25-a^2$ 3. m^2-64 4. $49-p^2$ 5. $4x^2-25$
 6. $16p^2-1$ 7. $9a^2-25x^2$ 8. $36c^2-49d^2$ 9. m^2-64a^2
 10. $16p^2-b^2$ 11. $a^2m^2-n^2$ 12. $p^2q^2-r^2$ 13. $a^2x^2-b^2y^2$
 14. $a^2b^2-c^2d^2$ 15. $4a^2b^2-c^2$ 16. $9x^2y^2-z^2$ 17. $9m^2n^2$
 $-16p^2q^2$ 18. $25p^2q^2-16r^2s^2$ 19. $a^2+b^2-c^2+2ab$
 20. $x^2-y^2-z^2+2yz$ 21. $4m^2+n^2-p^2+4mn$ 22. $16p^2$
 $+4q^2-r^2+16pq$ 23. $a^2+4b^2-9c^2+4ab$ 24. $4x^2+9y^2-16z^2$
 $-12xy$ 25. $9p^2+16q^2-25r^2-24pq$.

Exercise 20

1. 999999 2. 1689600 3. 1439879 4. $4xy-4xz$
 5. $4mn+4mp$ 6. $-5x^2+5y^2+24xy-8xz+40yz$
 7. $8ab-12ac$ 8. 74940 9. 576295 10. 331340
 11. 1975307 12. $(x+2y)(x-2y)$ 13. $(4a+c)(4a-c)$
 14. $(4m+5n)(4m-5n)$ 15. $(9p+10n)(9p-10n)$
 16. $(x+1)(x-1)$ 17. $(2+p)(2-p)$ 18. $(a+b+c+d)$
 $(a+b-c-d)$ 19. $(3x+2y)(x+4y)$ 20. $(a+b+1)(a+b-1)$
 21. $(x-2y+2)(x-2y-2)$ 22. 50^2-37^2 23. 265^2-56^2
 24. $(2x+3y)^2-(x-y)^2$ 25. $(4m-\frac{y}{2})^2-(m-\frac{5y}{2})^2$

Exercise 21

1. $4(2+x)$ 2. $3(a+2)$ 3. $2(a+2b)$ 4. $5(p-2q)$
 5. $a(b+c)$ 6. $x(y+z)$ 7. $2m(n+2p)$ 8. $xy(a-b)$
 9. $bc(a-d)$ 10. $xy(x+y)$ 11. $a^2(b-c)$ 12. $xyz(x+y+z)$
 13. $mn(a-b+c)$ 14. $p^2(a^2+b^2+c^2)$ 15. $a^2b^2(m^2+n^2+p^2)$.

Exercise 22

1. $(a+4)(a-4)$
2. $(2+m)(2-m)$
3. $(p+7)(p-7)$
4. $(9+c)(9-c)$
5. $(5m+4)(5m-4)$
6. $4(5+3d)(5-3d)$
7. $(1+3c)(1-3c)$
8. $(2a+3b)(2a-3b)$
9. $(4p+7q)(4p-7q)$
10. $(10x+11y)(10x-11y)$
11. $(ab+c)(ab-c)$
12. $(mn+2)(mn-2)$
13. $(5+abc)(5-abc)$
14. $(mn+pq)(mn-pq)$
15. $(3pq+4rs)(3pq-4rs)$
16. $(1+3abcd)(1-3abcd)$
17. $3(2a+3x)(2a-3x)$
18. $5(4x+5y)(4x-5y)$
19. $a(x+y)(x-y)$
20. $9(m+3n)(m-3n)$
21. $m^2(p+2q)(p-2q)$
22. $(x+y+2)(x+y-2)$
23. $(1+m+n)(1-m-n)$
24. $4ab$
25. $(10+2a+3b)(10-2a-3b)$
26. $(11x+4y)(4y-5x)$
27. $(7m+n)(m+5n)$
28. $(21c+8d)(9c+16d)$
29. $a(15x-14y)(x+14y)$
30. $(3a+b-2c)(a-3b)$
31. $(5x-7y+12z)(x-y+2z)$
32. $4xy(x^2+y^2)$
33. $(13m+5p)(5m+12n-11p)$
34. $4m(n-q)$
35. $8a(3b-4c+5d)$
36. $(a+b+c)(b+c-a)(c+a-b)(a+b-c)$
37. $(a+b-c+d)(a-b-c-d)$
38. $(xy-xz+yz-ay)(xy-xz-yz+ay)$
39. $(axy+bxy+apq-bpq)(axy+bxy-apq+bpq)$
40. $(a+b+c)(a+b-c)$
41. $(a+b-c)(a-b+c)$
42. $(x+y-2z)(x-y+2z)$
43. $(3m+n-p)(3m-n+p)$
44. $(c+a-b)(c-a+b)$
45. $(4x+2a+3b)(4x-2a-3b)$
46. $(a+b-1)(a-b+1)$
47. $(a-b)(a+b-2)$
48. $(a-b)(a+b-2c)$
49. $(ac-bd+ad+bc)(ac-bd-ad-bc)$
50. $(a+b+c+d)(a-b-c+d)$
51. $(a+b-c-d)(a-b+c-d)$

Exercise 23

1. $x-45=100$ (the number is x)
2. $x+67=200$ (the number is x)
3. $15x=375$ (the number is x)
4. $\frac{x}{12}=432$ (the number is x)
5. $\frac{x}{3}+\frac{x}{4}=35$ (the number is x)
6. $5x+20=100$ (the number is x)
7. $11x-63=190$ (the number is x)
8. $5x+15=6x$ (the number is x)
9. $x+(x+7)=117$ (number scored by Indrani is x)
10. $x+(x+8)+(x-5)=108$ (number of Baren's run is x)
11. $\frac{x}{2}+\frac{x}{4}+3=x$ (Sumanta

- went to market with Rs x) 12. $x + (x - 2) + \{(x - 2) - 3\} = 38$
 (Kamal's daily wage is Rs x) 13. $x + (x + 9) + (x - 9) = 156$
 (number scored by Ayesa is x) 14. $x - \frac{x}{3} = 1100$ (Mr.
 Friskin's pay is Rs x) 15. $5x + x = 42$ (son's age is x yrs)
 16. $2 \times 2 + 3 \times 2 + 3x = 28$ (quantity of rice is x kg)
 17. $x + \frac{x}{2} + \frac{x}{4} = 70$ (number of Tutun's marbles is x)
 18. $x = \frac{x}{6} + \frac{x}{8} + \frac{x}{4} + 165$ (total number of fruits is x)
 19. $3 \times 40 + 35x + 30(60 - x) = 2095$ (number of students of
 higher classes is x).

Exercise 24

1. 5 2. -9 3. 5 4. -16 5. 2 6. -5 7. 10
 8. 11 9. -2 10. 3 11. 3 12. 8 13. 3 14. $3\frac{2}{3}$
 15. -1 16. 2 17. 5 18. -3 19. 1 20. 20 21. 5
 22. $\frac{d-b}{a-c}$ 23. ab 24. ab 25. 13 26. 6 27. 145;

133; 25; 5184; 60; 16; 23; 15; Indrani scored 55, Manoara scored 62; number of Baren's run is 35; John's run is 43, Maidul's run is 30; Rs 12; Kamal's daily wage is Rs 15, that of Bansidhar is Rs 13 and of Haripada Rs 10; Ayesa scored 52, Nasifa scored 61 and Sayrah scored 43 marks; Rs 1650; Father's age is 35 yrs, son's age is 7 years; 6 kg; number of Tutun's marbles is 40, that of Tukai is 20 and of Tumpa is 10; 360; number of students in the higher classes is 35 and that in the lower classes is 25.

GEOMETRY



Chapter 1

Translation, Rotation and Reflection

1.1 Idea of Translation :

We always see around us various objects. Of them, some are in a state of rest, while others in a state of motion. We cannot think of any state of an object other than these two states. Trees, buildings, mountains, hills etc. are in a state of rest, while a moving train, bus, boat, steamer or man is in a state of motion. In general, we can say that if the position of an object does not change with time, we call such an object **static object**.

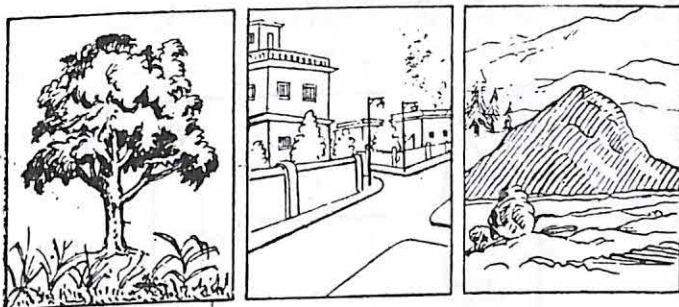


Fig. 1

On the other hand, if the position of an object changes with time, we call it a **moving** or **dynamic object**. The state of rest of a static object is called **equilibrium** and that of a moving object, the **motion**.



Fig. 2

1.2 Two types of motion :

The motion is mainly of two kinds, for example, translation and rotation. The motion in which the position changes but the direction does not change is called **translation**. Put your pencil on the table in such a way that one of its ends is in the north and the other, in the south. Move the pencil on the table to a certain extent so that the end of the northern side is directed to the north and that of the southern to the south.

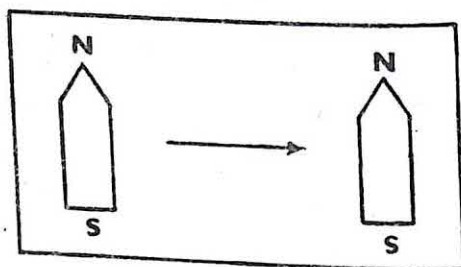


Fig. 3

In this motion of the pencil, the change of position has taken place but not the direction; this is what is called **translation** (motion). The motion in which the direction changes is called **rotation**.

As earlier, if the pencil is rotated in the direction opposite to what the hands of a clock move by pressing the northern end of the pencil, till the other-end points to the eastern direction then the motion under which the pencil comes from the previous state to the new state is called rotation. It is because of this motion the direction of the pencil has changed—earlier in the first state the pencil was spread north-south, now in the second state, it is west-east.

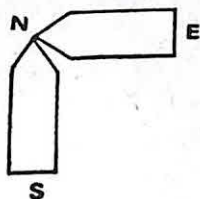


Fig. 4

1.3 Motion, Size and Shape of Solids :

One finds an interesting thing in the case of motion both ways. The size and shape of the pencil has not changed whatever be the motion, translation and rotation.

The pencil has remained that long or thick as it was earlier. If the pencil was round or sharp,

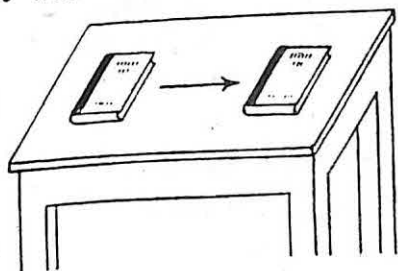


Fig. 5

it remains so. It is not just true for a pencil; it is true for any solid, for example, as in Fig. 5. It is shown that because of the translation of a book on a table,

none of the properties such as length, breadth, width or size of the book has not changed. Also in Fig. 6 the rotation of a cone is shown.

Even then the size of the cone has not changed.

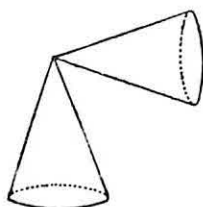


Fig. 6

1.4 Motion and Size and Shape of a Geometric Figure :

If on a plain paper we have a trace of a solid object we get a geometrical figure. We get a straight line by taking a similar trace along the length of the pencil, the trace of a book is rectangle, the trace of the dice of the Ludo is a square; the trace of the round lower portion of a cone is a circle. Therefore we can reasonably conclude—if the size or shape of a solid object does not change under translation or rotation then the geometrical figure which is the trace of a solid object does not also change in size or shape under translation or rotation.

In this chapter, this will be clearly demonstrated with the help of different handi-work and experiments.

1.5 The size and shape of a triangle is unchanged under translation :

Experiment—1. Take a piece of paper on the table, put on it a triangular piece of wood. Now mark on the paper with the help of a pencil along

the sides of the triangle. Suppose ABC is the first position of the triangle.

Now put the scale along the side BC of the wooden triangle ABC . Press the scale with one hand and move the triangle to the right along the scale with other hand. Take care, that the side BC remains always attached to the edge of the scale during motion. The motion that the triangle has is translation. There is a change in the position of

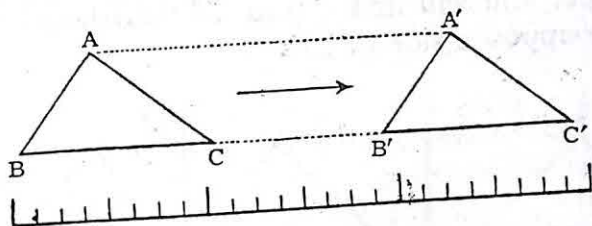


Fig. 7

the triangle but no change in the direction has taken place. The side BC is in the same direction as it was. Now mark with the pencil the sides of the triangle in the new position. Suppose in the new position the triangle is $A'B'C'$. Now observe by measuring with the scale that $AB = A'B'$; $BC = B'C'$, $AC = A'C'$.

The size and the shape of triangle has not changed under translation. Here, under translation, the positions of points A , B , C have become A' , B' , C' respectively.

Observe by measuring $AA' = BB' = CC'$. AA' , BB' , CC' are called lines of translation.

Note : The lines of translation of different points of the same object are parallel. The magnitude of translation is also same or in other

words, the length of the line-segments AA' , BB' , CC' is same.

You can verify the truth of the above by experimenting with the size of any object.

1.6 Experiment by Reflection or Paper folding :

Draw a triangle ABC on a piece of paper (Fig. 8) and after inking the sides and folding it along the straight line EF press well the two parts. Opening the paper you will find a trace of the triangle ABC on the opposite side of EF .

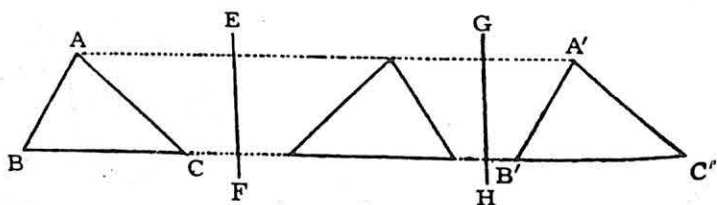


Fig. 8

This way of displacement of a figure is called reflection. The straight line along which the fold has taken place is called the line of reflection. Here the straight line EF is the line of reflection.

Again fold the paper along the straight line GH drawn parallel to EF and press the part of the second position on the vacant part.

Open the fold and you will find a trace of the triangle on the new part. Name this triangle $A'B'C'$. So after two successive reflections on EF and GH , $\triangle ABC$ has come to the position $\triangle A'B'C'$. Observe that a translation along AA' of magnitude equal to the length of the segment AA' would have displaced $\triangle ABC$ to the same new position $\triangle A'B'C'$. Now verify by measuring with

a scale that $AB = A'B'$, $BC = B'C'$, $AC = A'C'$. Therefore, the size or the shape of the $\triangle ABC$ has not changed under translation.

Note : You learn from the above the additional fact that successive reflections in two parallel lines are equivalent to a translation. You can examine the translation of quadrilaterals and polygons.

1.7 Properties of Translation:

From various experiments on translation, we can arrive at the following conclusions :

- (i) Under translation every point of an object moves in the same direction.
- (ii) Under translation there is an equal displacement for every point in the same direction.
- (iii) Under translation there is no fixed point.
- (iv) All straight lines in the direction of the translation are fixed straight lines.
- (v) A point between any two points remains so in the position after translation.
- (vi) The size and shape of an object do not change under reflection, the orientation is also unchanged.
- (vii) Under translation the base-plane of the object is not changed.
- (viii) Under translation the corresponding distances between different points remain unchanged.
- (ix) Under translation collinear points remain collinear.
- (x) Under translation the old and new positions of a straight line are parallel, except for

a straight line in the direction of the translation itself.

- (xi) Under translation the magnitude or the orientation of an angle is not changed.

Perform suitable experiments yourselves to verify the properties of translation.

Remark : Straight lines which do not change their positions under translation are called fixed straight lines or invariant with respect to translation.

* 1.8 Properties of Reflection :

Verify the following properties of reflection by folding a piece of paper once along a straight line.

- (i) The points on the line of reflection are fixed points.
- (ii) The line of reflection is a fixed straight line.
- (iii) The join of a point and its reflection is a fixed straight line and this straight line is perpendicular to the line of reflection.
- (iv) A point and its reflection are equidistant from the line of reflection.
- (v) A point between any two points remains so in its new position after reflection.
- (vi) Under reflection collinear points remain collinear.
- (vii) Under reflection the distance between any two points remains unchanged.
- (viii) Under reflection the magnitude of an angle remains unchanged but there is a change in orientation.
- (ix) Under reflection the size and shape of a plane figure do not change but there is a change in orientation.

Remark : The points and the straight lines which do not change under reflection are called fixed points and fixed straight lines respectively with respect to reflection.

Exercise 1

1. What do you mean by rest and motion? Describe with examples.

2. What is translation? Explain translation with the help of simple examples and figures.

3. Demonstrate the translation of a triangle by drawing a triangle and reflecting it in two parallel lines. Call it ABC and $A'B'C'$ in the two positions of the triangle. Examine by measurement if $AB=A'B'$, $BC=B'C'$, $CA=C'A'$. What will be the mutual relation between AB and $A'B'$ or CA and $C'A'$. Examine by measurement if the magnitude of an angle of a triangle is equal to the magnitude of the same angle under reflection.

4. Of the following statements, mark those with the sign ($\checkmark$) which are correct and with this sign ($\times$) which are incorrect, write them after correction.

- (i) Under translation there is displacement of every point in the same direction.
- (ii) Only one reflection of an object produces translation.
- (iii) A straight line is parallel to its new position under translation.
- (iv) Under translation collinear points do not remain collinear.
- (v) Under translation the magnitude of an angle as well as its orientation remains unchanged.
- (vi) Under translation all points remain fixed.
- (vii) Under translation mutual distances between different points do not change.
- (viii) Under translation there are fixed points but not fixed straight lines.
- (ix) Under translation the shape of an object does not change.
- (x) Under translation the base plane of an object does not change.
- (xi) Under translation displacement of different points of an object are different in magnitude.
- (xii) All points on the line of reflection are fixed.

1.9 Size and shape of a solid object do not change under rotation :

You have seen that this kind of change does not take place under translation. Let us observe with the help of one or two simple experiments that the shape and size of any solid object do not change under rotation.

1st Experiment : A small but heavy object or a piece of stone tied with a thread and rotated with the help of a finger revolves in a circular path.

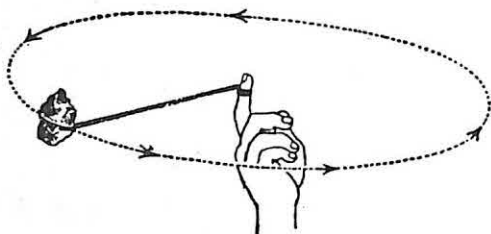


Fig. 9

You can perform this experiment on your own. The change of position of the object in this case is called the rotation, for the direction of the object is always changing under this motion.

2nd Experiment : Fasten a strong rope to the branch of a tree. The length of the rope should be such that its other end does not touch the ground but remains well above it. Fasten tightly to the lower end of the rope a brick with one of its sides coloured black. Pull the brick through some distance on one side and then leave it. You will find the brick will slowly come to a stop after swinging from one side to another for a while. So long as

the swing continues the black face of the brick will be changing its side, now downward, now somewhat inclined to the left and then a little more inclined to the right; and not the black face alone the entire brick with its black face is under rotation. It should be mentioned here that in this case, there is a translation of the brick also. It is possible for an object to have two kinds of motion at the same time. But here we are talking particularly about rotation, for that is the subject of our present discussion. Thus it is clear that there has been a considerable extent of rotation of the brick.

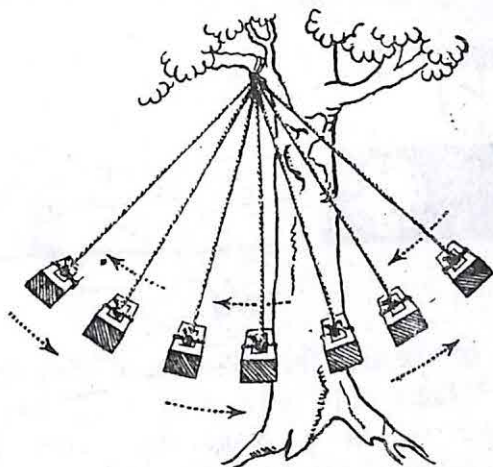


Fig. 10

Now we have to compare any two positions of the brick during rotation. Let us suppose, the position before the brick was pulled across was its first position and any position after that is the second position. Catch hold of the brick when it comes to its second position and untying it from the rope

measure the length, breadth, width of the brick you will find that there has been no change in these quantities since brick was in its first position. Also there may have been some peculiarities in the shape of the brick such as a hollow or one of its faces or the inscribed initials of the name of the brick maker. If there had been any such peculiarity at the beginning it remains unaltered after the rotation. In other words we can say that the size or the shape has not changed under rotation in any respect.

In our daily life, we observe rotation of various objects. For example, the rotation of the wheel of the can, the rotation of the hands of a clock, the

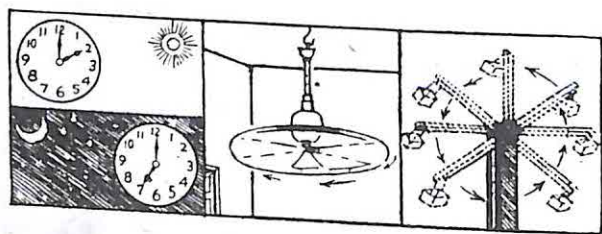


Fig. 11

rotation of electric fan. Some of you must have sat in a 'Nagar-dola' you must have observed that when you sit in a Nagar-dola that it moves continuously in the form of a circle. If you observe a definite part of the ground from swing, then you must have found that the swing crosses that portion of the ground after a definite time interval. Thus it is clear that there is a change of position of the swing at every point, but the swing comes back to its earlier position after a definite time interval. This kind of motion of object is rotation (motion).

[The revolution of planets like earth round the sun and the revolution of the moon about the sun are all examples of rotation in nature. It is not the solar system alone, you will come to know after growing up, that rotation exists even in smallest objects. The atoms in a molecule are revolving continuously round a centre.]

1.10 Geometric idea of rotation :

Take a triangular piece OAB of wood and fasten the point O in the place with a needle such that OAB can rotate about the point O . Mark with pencil the plane around AOB . If the side OA is slightly

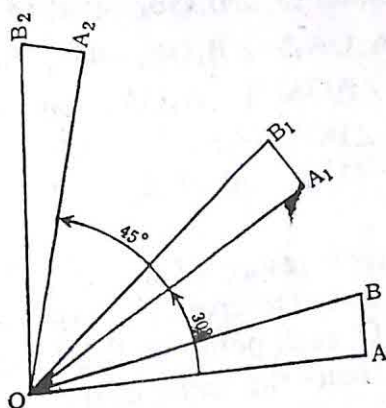


Fig. 12

pushed, the triangle OAB is rotated in the direction opposite to the hands of a clock. Suppose the triangle OAB after rotation through 30° comes to the position OA_1B_1 , mark the sides of OA_1B_1 by the pencil. Push one side exactly in the same way. Suppose it comes to the position OA_2B_2 after rotation through angle 45° . Mark with pencil, the triangle OA_2B_2 . The point O about which OAB rotates

called the **centre of rotation**. The angle (magnitude) through which the object rotates is called **revolution or rotation**. In the first case, the angle of rotation OAB is 30° and in the second 45° . After the first rotation (i.e. after rotation to 30°) the position of OA is OA_1 , and that of OB is OB_1 . Therefore, it can be said that OA and OB come to the positions OA_1 and OB_1 respectively through a rotation of angle 30° . Measure with the protractor and verify that $\angle AOA_1$ equal to $\angle BOB_1$ equal to 30° . Likewise, after the second rotation 45° (angle 45°) OA_1 , OB_1 come to the position OA_2 , OB_2 respectively. Measure with the protractor and verify that $\angle A_1OA_2$ equal to $\angle B_1OB_2$ equal to 45° .

Hence $\angle A_1OA_2 = \angle B_1OB_2 = 45^\circ$; Hence,
 $\angle AOA_2 = \angle AOA_1 + \angle A_1OA_2 = 30^\circ + 45^\circ = 75^\circ$.
 Similarly, $\angle BOB_2 = 75^\circ$. Verify with a scale,
 $OA = OA_1 = OA_2$; $OB = OB_1 = OB_2$; $AB = A_1B_1 = A_2B_2$.

No change in respect of size or shape of the object has taken place. Observe that, other than the centre of rotation O , each point of OAB has suffered displacement. Hence the centre of rotation with respect to rotation is a fixed point.

OAB , OA_1B_1 and OA_2B_2 coincide with each other completely.

1.11 Operation of rotation with the help of reflection or paper folding :

That the displacement which takes place in a plane after two successive reflections can be shown to be a rotation with simple experiments.

Experiment : In the plane two straight lines OP and OQ intersect at the point O. Suppose that any point A reflected first in the line OP becomes A_1 . Again after reflection in line OQ it becomes A_2 . [On the paper the point A is marked with ink-dot. Then folding in two straight lines OP and OQ there will be the mark of ink-dot at A_2 , the final position.] In

other words, A_2 is the final position of A. Measure with the help of a scale to find that $OA = OA_1 = OA_2$; therefore the circle drawn with O as centre and OA as radius must pass through the

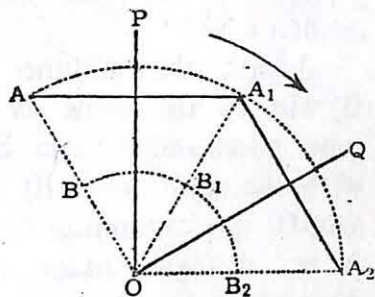


Fig. 13

points A_1 and A_2 (show it by the circle). The position of A after rotation with respect to the centre of rotation will be on that circle. Observe further that angle of rotation is angle AOA_2 . Measure with the protractor to see that angle AOA_2 is equal to $2\angle POQ$.

Hence, it can be said, whatever be the angle between two intersecting lines, the angle of rotation is twice the angle between them.

Note: Since $OA = OA_1 = OA_2$, it can be said that under rotation the point at any position maintains equal distance from the centre of rotation.

Whatever be the point on OA, it will be on OA_2 after rotation. Therefore, it can be said that the

property of collinearity of collinear points is not changed under rotation. Over and above this, every point of an object or a geometrical figure is rotated through the same angle of rotation. In the Fig. 13; the point A is rotated through $\angle AOA_2$. The point B on OA is rotated through the $\angle BOB_2$. Since B_2 must lie on OA_2 we must have $\angle BOB_2 = \angle AOA_2$. This can also be shown for any other point on OA.

Again, the distance between points A and B will be the same as the distance between the new positions A_2 and B_2 after rotation. Measure with the scale to verify that $AB = A_2B_2$. In other words, corresponding distance between two points is not changed under rotation. Besides this, any point between A and B will be between A_2 and B_2 after rotation.

1.12 The magnitude of the angle of rotation or its orientation does not change :

You have seen that in the triangular object AOB comes to the position A_1OB_1 after rotation through an angle 30° first and secondly to the position A_2OB_2 through the rotation of an angle of 45° . Measure with the protractor and verify that $\angle AOB = \angle A_1OB_1 = \angle A_2OB_2$. You understand easily from this that under rotation the orientation of the angle and its magnitude remain unchanged.

1.13 Experiments on rotation of a triangle with the help of reflection or paper folding :

On a piece of plain paper take two intersecting straight lines OP, OQ. Take a triangle ABC on the

side opposite to the side where OQ lies on the plane OP , its sides being marked with ink along their lengths. Now fold the paper along the straight line OP so that there will be trace of the triangle on the side of OQ . Let us call it $A_1B_1C_1$. This is the reflection of the triangle ABC in the line OP . Similarly, the reflection of $A_1B_1C_1$ in line OQ is $A_2B_2C_2$.

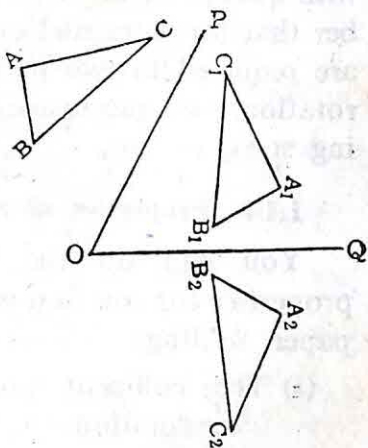


Fig. 14

Now, join OA , OB , OC , OA_2 , OB_2 , OC_2 and

measure with the help of a protractor to see that $\angle AOA_2 = \angle BOB_2 = \angle COC_2 = \theta$ (say) [θ is a letter of greek alphabet, its pronunciation is 'Theta'.]

Therefore, we can say that by rotating the triangle ABC about the point O in the clockwise direction through an angle θ , we get the triangle $A_2B_2C_2$. In other words, the triangle ABC will coincide with the triangle $A_2B_2C_2$ in all respects. Measuring with the scale see that $A_2B_2 = AB$, $B_2C_2 = BC$, $C_2A_2 = CA$ and with the help of protractor, see that $\angle A_2B_2C_2 = \angle ABC$; $\angle B_2C_2A_2 = \angle BCA$ and $\angle C_2A_2B_2 = \angle CAB$.

In other words, as in the case of translation, in the case of rotation the size and shape of the triangle remain unchanged.

The statement is not applicable for triangle alone but is true for any geometrical figure. You can well understand this truth after doing similar experiments

with quadrilateral, circle etc. But you must remember that for a translation, two successive reflections are required in two parallel straight lines, and for rotation, two successive reflections in two intersecting straight lines.

1.14 Properties of rotation :

You can do the following experiments on properties for rotation with the help of reflection or paper folding:

- (i) The collinear points will remain collinear after rotation.
- (ii) A point between any two points remain between them after rotation.
- (iii) The corresponding distances between different points do not change after rotation.
- (iv) The magnitude of an angle or its orientation does not change under rotation. [It has to be made clear that the orientation of an angle changes after one reflection even though its magnitude remains unchanged. In the second reflection this change is restored.]
- (v) For every point of a plane figure, there is rotation in the same direction.
- (vi) The magnitude of the angle of rotation of every point is the same.
- (vii) All points except the centre of rotation change in positions under rotation. So, the centre of rotation is a fixed point.
- (viii) There is no fixed line under rotation.

Exercise 2

1. What is rotation ? Give some examples of rotation that are commonly seen in daily life.
2. What is centre of rotation and angle of rotation ? Describe geometrically with the help of figure.
3. What is the main difference among reflection, translation and rotation ? State this with the help of a figure.
4. Show with the help of experiments that successive reflections in two straight lines result in rotation.
5. Show with the help of a diagram that there is no change in magnitude or orientation of the angle under rotation.
6. Show translation and rotation with the help of reflection or paper folding.
7. Of the following statements mark ($\checkmark$) those which are correct and ($\times$) that are incorrect and rewrite them after correcting the errors :
 - (i) Under rotation, there is a fixed point.
 - (ii) Under rotation, the orientation of an angle does not change.
 - (iii) Under rotation, the size of an object does not change.
 - (iv) There results a rotation under two successive reflections in two parallel lines.
 - (v) Under rotation, the property of collinearity of points is not preserved.
 - (vi) Under rotation of an object, the angle of rotation of every point remains same.
 - (vii) The corresponding distances between points under rotation change.
 - (viii) Under rotation, all points suffer change of position.
 - (ix) Under rotation, there is no fixed point.
 - (x) Under rotation, the orientation of angle does not change.
 - (xi) Under reflection, the orientation of an angle does not change.
 - (xii) There results translation from successive reflections in two intersecting straight lines.
 - (xiii) Under reflection, the property of collinearity of some points is not preserved.
 - (xiv) Under rotation, the corresponding distances between points remain unchanged.
 - (xv) All points suffer displacement under reflection.
8. In the following figures, state how fig. 2 can be obtained from fig. 1 by one or more than one transformations. [Since we get geometrical figures in new form with the help of transla-

tion, rotation or reflection, we call such operations as transformation and the figures obtained are called transformed figures]. Find also the line of reflection in the case of reflection, line of displacement in the case of translation and centre of rotation in the case of rotation.

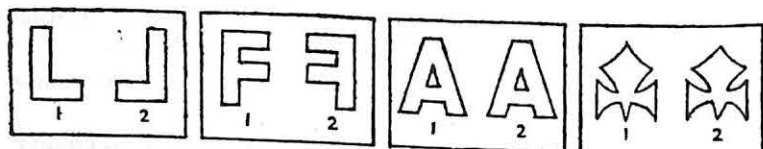


Fig. 15

9. Show with the help of reflection or paper folding that:
- (i) The length of a line-segment does not change under reflection but there is a change in orientation.
 - (ii) Under translation, neither the length of a line-segment nor orientation changes.
 - (iii) Under rotation, neither the length of the line-segment nor orientation changes.
 - (iv) The size or shape of a triangle does not change under reflection but there is a change in orientation.
 - (v) The size, shape and orientation of a triangle does not change under translation.
 - (vi) Under rotation the size, shape and orientation of a triangle does not change.
 - (vii) Under reflection, the size or shape of a quadrilateral does not change but orientation changes.
 - (viii) The size, shape and orientation of a quadrilateral do not change under translation.
 - (ix) The size, shape and orientation of a quadrilateral do not change under rotation.
 - (x) The size or shape of a triangle does not change under reflection but its orientation changes.
 - (xi) The size, shape, orientation of a circle do not change under translation.
 - (xii) The size, shape, orientation of a circle do not change under rotation.

Chapter 2

Verification of Some Geometrical Properties

2.1 One of the objectives of the study of geometry is to learn geometrical properties of different types of plane figure. In this chapter some properties of Translation, Rotation and Reflection will be verified.

2.2 Corresponding angle :

Let us suppose that the straight line EF intersects the lines AB and CD in points G and H respectively (Fig. 16). Then we shall call each one angle of the following pairs of angles to be corresponding angle of the other.

($\angle EGB$, $\angle GHD$), ($\angle AGE$, $\angle CHG$), ($\angle CHF$, $\angle AGH$), ($\angle FHD$, $\angle HGB$).

2.3 When a straight line intersects any other two straight lines then those two straight lines will be parallel to each other if a pair of corresponding angles are equal.

Let us suppose that the straight line EF intersect AB and CD in points G and H respectively and one pair of corresponding angles $\angle EGB$ and $\angle GHD$ are equal. Verify that AB and CD are parallel to each other.

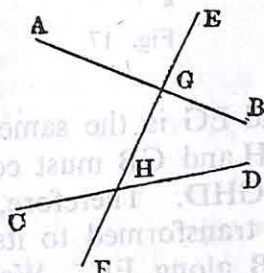


Fig. 16

Verification with the help of Translation : In fig 17, cut out the $\angle EGB$ after putting a piece of paper on fig 17. The point G on the paper is placed on the point H of the figure and then put

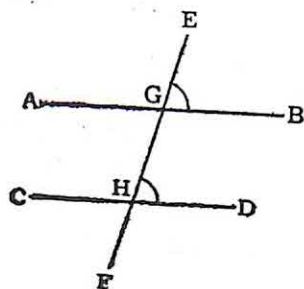


Fig. 17

the edge GE along edge HG. You will find that the edge GB has coincided with edge HD. What has been done here is the operation of Translation. In other words the point G is brought to be point H after translating $\angle EGB$ along the line EF. Since EF, GH

and EG is the same straight line, EG will fall on GH and GB must coincide with HD for $\angle EGB = \angle GHD$. Therefore, we can say that the line GB is transformed to its new position HD by moving GB along EF. We have seen earlier that under translation any straight line in the new position will be parallel to its earlier position except the straight line along the direction of translation. Therefore GB and HD are parallel to each other. Since GB is a part of AB and HD that of CD then the two lines AB and CD will be parallel to each other as a whole.

2.4 Two intersecting straight lines can not both be parallel to a third straight line.

Let us suppose that the two intersecting straight lines AB and CD meet at a point G. It has to be verified that both AB and CD cannot be parallel to a third line.

Verification with the help of Translation : Let us suppose that the line AB is parallel to the third line EF . It is to be shown that the line CD is not parallel to the line EF . We know that except the straight line in the direction of translation any other straight line in the new and the first position is parallel under translation and any point on the line will continue to be on the line in the new position. We can therefore say that AB and EF are parallel. Then let us carry out translation in such a way that the point G on AB is displaced along the line CD and when the new position of AB merges with EF then also the new position of G is a point on the straight line EF . Let us recall the point G' . In the first position i.e. before completion of translation the point G is not merely on AB but also on the line CD and we know that under translation this property will not be changed at all. In other words the point G will lie on CD . Therefore G' is the point of intersection of the two lines CD and EF , the new position of AB . In other words CD and EF cannot be parallel because of being intersecting.

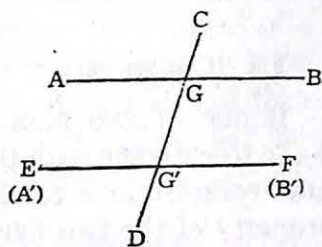


Fig. 18

Note: You can verify yourselves this geometric information by cutting a piece of paper. Trace the whole figure by placing a piece of this paper. Now cut out the upper portion by moving along AB . Now place the loose papers on the geometrical figures in such a way that the side AB is with EF and

G with G' . You will find that the portion CG is placed along CD . This means that CG , namely, the whole of CD intersects the line EF at the point G' . CD and EF cannot be parallel, because of their mutual intersection.

2.5 Congruence :

If one of two plane geometrical figures can be made to coincide with the other by means of translation, rotation or a combination of the two then this property of the two figures is called congruence and the two figures are called congruent figures. We express the congruence by the symbol $\cong$.

Remark : If the two figures are of opposite orientation then we are to make use of reflection.

We shall now discuss congruence of two triangles.

2.6 SAS formula :

Let us suppose that between two triangles ABC

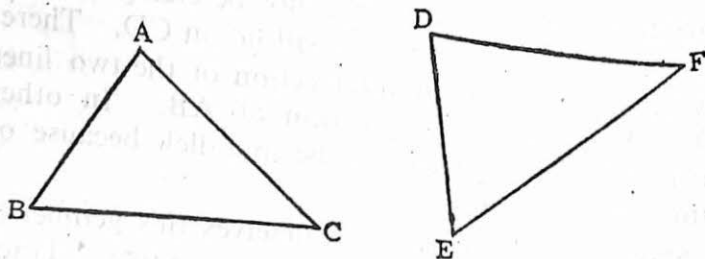


Fig. 19

and DEF , $AB=DE$, $AC=DF$ and the included $\angle BAC = \angle EDF$.

We are to verify that $\triangle ABC \cong \triangle DEF$.

Verification : First this property of congruence will be shown by simple experiment. Cut out a piece of paper on the measure of triangle ABC. Now place the triangular piece of paper on the triangle DEF such that the point A falls on point D. Fixing up the point A on the point D by means of a pin move the piece of paper both ways such that the side AB is placed on side DE. You will find then that the trace of the triangle ABC on the paper has coincided completely with the triangle DEF.

Remark 1. You can easily carryout this experiment on your own but it will be helpful for the work if you are careful about a certain thing. After having obtained rightly the trace of the triangle on a piece of paper cutout the paper of a bit larger size. Then both the fixing of the pin at A on D and moving the paper will be easier.

Remark 2. In order to be completely sure about the above properties of congruence there should be one or two more experiments. You will find that of the three given conditions ($AB=DE$, $AC=DF$, $\angle BAC = \angle EDF$) if one is altered the

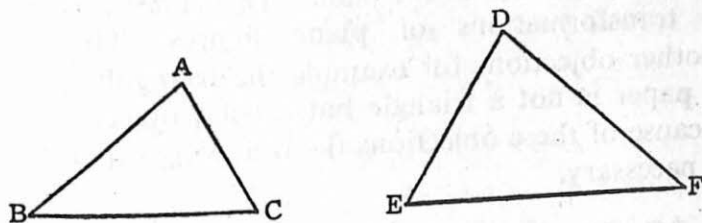


Fig. 20

two triangles cannot be made to coincide despite the above experiment. For example, if $AC < DF$ instead

of $AC = DF$ then the displaced triangle ABC will have its position as in fig. 21.

If $AB < DE$ then the position after displacement will be as in fig. 22.

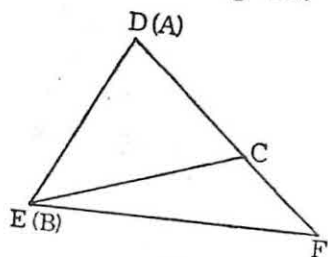


Fig. 21

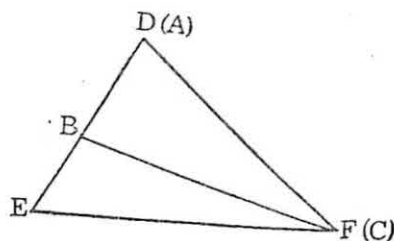


Fig. 22

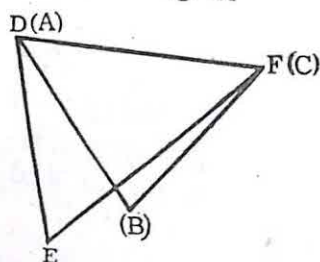


Fig. 23

Again if $\angle BAC < \angle EDF$ then the position after displacement will be as in fig. 23.

Remark 3 : The truth of formula SAS used to be demonstrated

with the help of the above procedure. In recent time some objections have come up saying that in this procedure one has to take out the triangular piece of paper out of the plane. This is not expected of transformations of plane figures. There is another objection; for example the triangular piece of paper is not a triangle but a solid object. It is because of these objections the following experiment is necessary.

2.7 Verification of SAS formula with the help of translation and rotation :

As before between two triangles ABC and DEF;
 $AB = DE$, $AC = DF$, $\angle BAC = \angle EDF$ (Fig. 19).

It is to be verified that the two triangles are congruent.

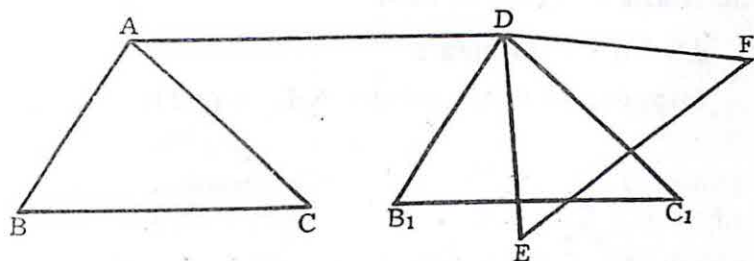


Fig. 24

Experiment : Join AD. Translate the triangle ABC along AD through the magnitude AD. The point A will then coincide with point D. But it is difficult to carry out this translation correctly. The point A may move this way or that way because of the operation on triangle ABC. If this be the case then it will be a combination of translation and rotation. But there is nothing to loose even for this. Let us suppose that the displaced form of the triangle ABC, because of this kind of pure translation or a combination of transformations is DB_1C_1 . Now rotate the triangle DB_1C_1 about D as centre through $\angle B_1DE$ in the direction opposite to the hands of the clock. You will find that the triangle DB_1C_1 or the triangle ABC has coincided with the $\triangle DEF$ completely.

Observation : If there is an exception in respect of any one of the given conditions you will find that even in this experiment the two triangles can not be made to coincide with each other completely.

Remark : In the formula SAS, S=Side, A=Angle. From the letter A between two S one has

to take the angle to be the included angle or in other words if two sides and the included angle be equal the triangles are congruent.

2.8 AAS formula :

Suppose in two triangles ABC and DEF:

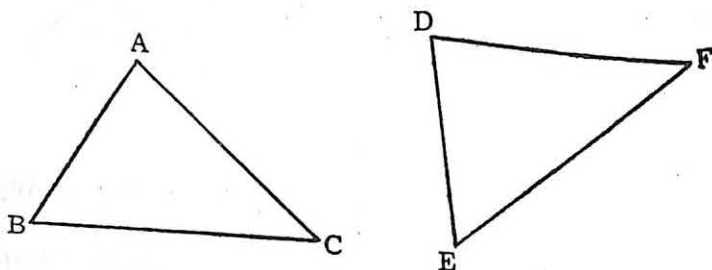


Fig. 25

$\angle ABC = \angle DEF$, $\angle BCA = \angle EFD$ and $AB = DE$.

It is to be verified that $\triangle ABC \cong \triangle DEF$.

2.9 Verification of AAS formula with the help of translation and rotation :

Cut out a piece of paper after obtaining a trace of $\triangle ABC$ on the paper placed on triangle ABC.

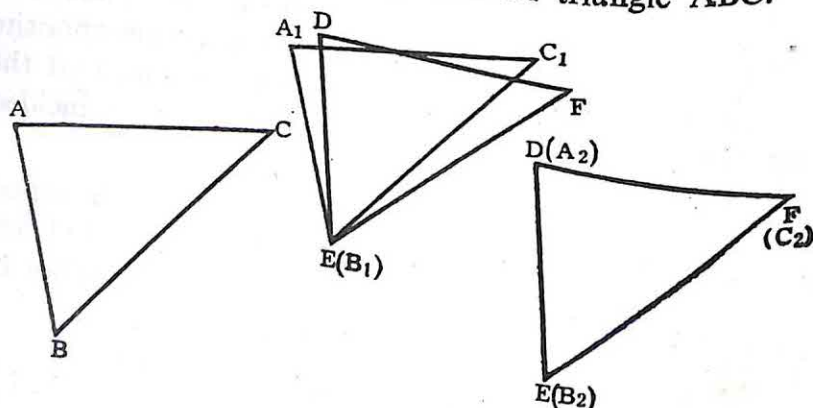


Fig. 26

Then carry out the translation of the triangular piece of paper from B to E by means of a scale. The translation is to be carried out carefully such that the piece of paper is not rotated. Of course it will not matter much if it rotates a bit. The point B will coincide with the point E because of this translation. The new position of $\triangle ABC$ is $\triangle A_1EC_1$ or $\triangle A_1B_1C_1$. Now rotate about B_1 or E as centre of rotation $\triangle A_1EC_1$ or $\triangle A_1B_1C_1$ through the $\angle A_1ED$ in the direction of the hands of the clock. You will find that in the new position the triangular piece of paper has coincided completely with DEF. In other words the two triangles are congruent or $\triangle ABC \cong \triangle DEF$.

Remark : It is to be pointed out that of the three given conditions if there be an exception even in respect of one the two triangles can not be made to coincide even with the same kind of translation-rotation.

If $\angle ABC > \angle DEF$, then the final position after translation-rotation will be as in fig. 27. If $\angle ABC < \angle DEF$, it will be as in fig. 28.

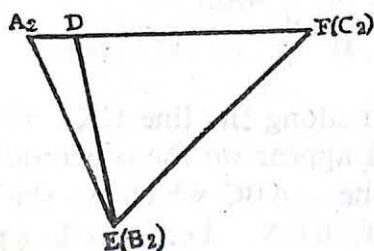


Fig. 27

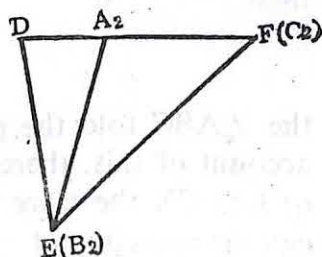


Fig. 28

If $BC < EF$, it will be as in fig. 29 and $BC > EF$ it will be as fig. 30.

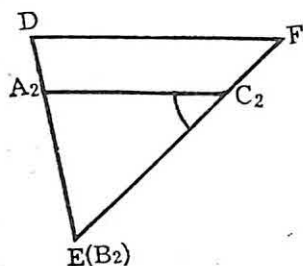


Fig. 29

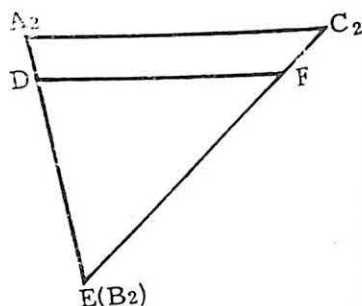


Fig. 30

2.10 Verification of formula SAS, AAS with the help of reflection or paper folding :

Draw on a piece of paper triangles ABC and DEF such that BC and the side EF are on the same straight line. Now draw on the line $BCEF$ the perpendicular CX at the point C . Now after inking the sides of

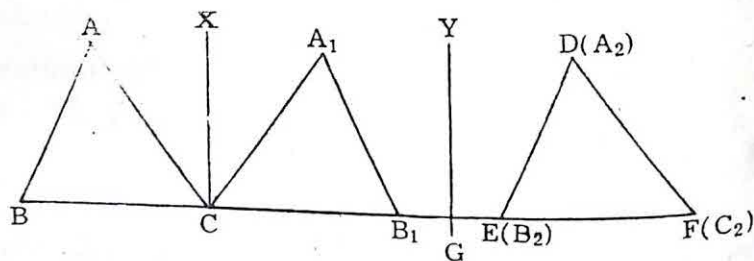


Fig. 31

the $\triangle ABC$ fold the paper along the line CX . On account of this, there will appear on the other side of line CX the trace of the $\triangle ABC$ which we shall call the reflection of $\triangle ABC$ in CX . Let the reflected figure be A_1B_1C . The point B_1 will be collinear with $BCEF$.

Now bisect the line segment B_1E at G . The bisector GY is perpendicular to $BCEF$. Inking the sides of $\triangle A_1B_1C$ and then folding the paper along the line GY you will find that trace of $\triangle A_1B_1C$ has coincided with $\triangle DEF$. Call this trace $\triangle A_2B_2C_2$. This is the result of reflection of $\triangle A_1B_1C$ in straight line GY and translation of the original triangle ABC along $BCEF$ and through the distance BE . This translation or reflection or successive reflection in two parallel lines makes $\triangle ABC$ coincide with $\triangle DEF$ or

$$\triangle ABC \cong \triangle DEF.$$

N.B. 1. This procedure is true equally for both SAS and AAS formulae.

2. In this section a particular position of the two triangles has been chosen. But the use of reflection can be made for any position. If the two triangles have the same orientation, then we have to use two reflections and if opposite, one or three reflections.

2.11 Congruence of Circles :

Of two circles if one is made to coincide with the other by translation, rotation or reflection, the two circles are called congruent. It is easily seen that the two congruent circles have equal radii.

2.12 In congruent circles (or in the same circle) equal chords cut off equal arcs and they subtend equal angles at the centre.

(i) Verification using rotation (in the same circle) :

Suppose O is the centre of a circle and AB and CD are two equal chords of this circle. We shall verify using rotation that two arcs AB and CD are equal in lengths and AB and CD subtend equal angles at the centre or $\angle AOB = \angle COD$.

Join OA , OB , OC , OD , then two triangles OAB and OCD are formed. Now with the centre O of the circle as centre of rotation rotate the figure through $\angle AOC$ opposite to the direction of the hands of clock (fig. 32).

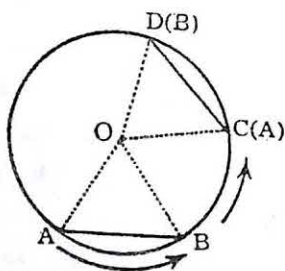


Fig. 32

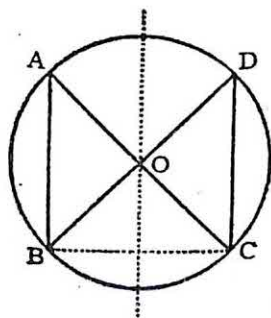


Fig. 33

Then you will see $\triangle OAB$ coincides completely with $\triangle OCD$ and therefore arc $AB = \text{arc } CD$ and $\angle AOB = \angle COD$. We wanted to show these two facts.

(ii) Verification using reflection (in the same circle) :

In the circle with O as centre $AB = DC$. We shall show using reflection that arc $AB = \text{arc } DC$ and $\angle AOB = \angle DOC$.

Drawing the Fig. 33, on oiled paper fold it about O such that the point B falls on point C (the line of fold BC is perpendicular bisector of the segment BC) then you will find that the point A falls on the point D and one part of the circle coincides with its other part. In other words the arc AB coincides with arc DC and $\angle AOB$ with $\angle DOC$ by reflection. Hence we can say $\text{arc AB} = \text{arc DC}$ and $\angle AOB = \angle DOC$.

N.B. If the given condition $AB = DC$ is changed then none of the above two facts will be true.

In fig. 34, $AB < DC$.

Now if with O as centre of rotation, the figure is rotated through $\angle AOD$ opposite to the hands of the clock then the new position of $\triangle OAB$ will be $\triangle OA_1B_1$ and it will be found that $\text{arc AB} = \text{arc } A_1B_1 < \text{arc DC}$ and $\angle AOB = \angle A_1OB_1 < \angle COD$.

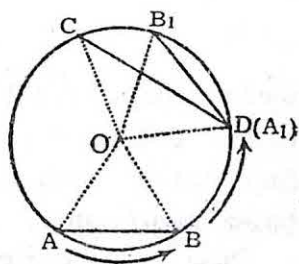


Fig. 34

(iii) Verification using composition of translation and rotation (Congruent circles) :

Let us suppose that in two congruent circles with respective centres O and O'. AB and CD are two equal chords in the circles with O and O' as centres respectively.

It is to be verified that $\text{arc AB} = \text{arc CD}$ and $\angle AOB = \angle CO'D$.

Join OO' . On a piece of paper trace the first circle with OA, OB and AB. Cut out the circular

piece of paper along the circumference. Now place your scale in such a way that it touches from below the two circle at two points. With a little effort you can place the scale in such a way that the circular

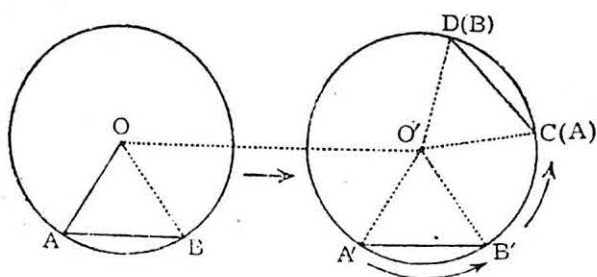


Fig. 35

piece of paper can be put on the first circle in order that two figures coincide. As a result you will find that the scale has touched the circular piece of paper at one point only.

Now move the circular piece of paper carefully along OO' such that the point O of the paper coincides with the point O' . It is to be observed during this motion that the paper does always touch the circle. The point of contact of the paper should always remain the same and the change in position resulting from this will always be a pure translation. To be careful about it, it is good that one should mark with a ink dot the point of contact of the paper with the scale in its first position. The new position of $\triangle AOB$ under translation is $\triangle O'A'B'$. Now rotate with the point O' as the centre of rotation the figure through $\angle A'O'C$ opposite to the hands of the clock so that $\triangle O'A'B'$ coincides with $\triangle O'CD$ completely; in other words the chord AB of

the first circle coincides with the chord CD of the second circle. Therefore arc $AB = \text{arc } CD$ and $\angle AOB = \angle CO'D$.

(iv) Verification using reflection (Congruent circles) :

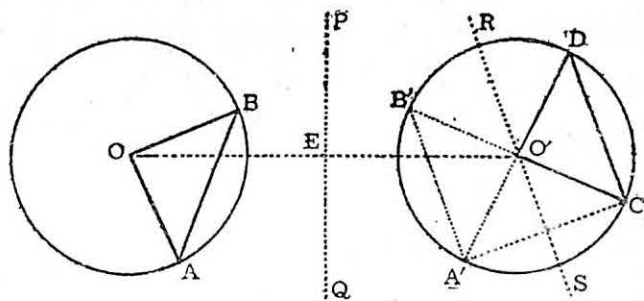


Fig. 36

Two circles with O and O' as centres are congruent. Chord AB of the circle with O as centre = chord CD of the circle with O' as centre. Using reflection we shall verify that arc $AB = \text{arc } CD$ and $\angle AOB = \angle CO'D$. Drawing the above figure (fig. 36) on an oiled paper reflect in the line PQ , the perpendicular bisector of the segment OO' (fold along PQ). You will find that the circle O as centre coincides with circle O' as centre. Let us take the reflection of the points A and B as A' and B' respectively. Now reflecting along RS , the perpendicular bisector of the segment $A'C$ (folding along RS for the second time), it can be found that the point A' coincides with C and B' with D . Therefore we can say arc $AB = \text{arc } A'B' = \text{arc } CD$ and $\angle AOB = \angle A'O'B' = \angle CO'D$.

Remark : If any one of the given conditions is violated the two facts will not be true. Look at the following figure.

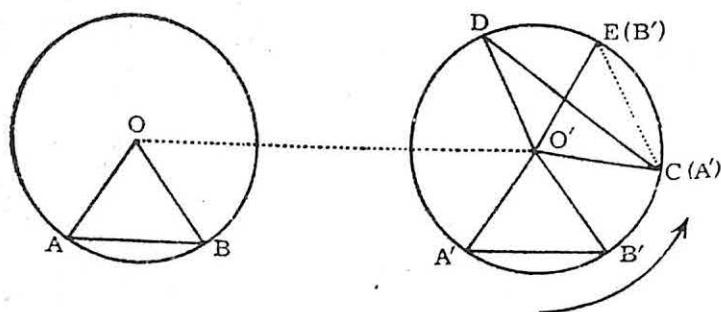


Fig. 37

Here $AB < CD$ and therefore

$\text{arc } AB = \text{arc } A'B' = \text{arc } CE < \text{arc } CD$ and
 $\angle AOB = \angle A'O'B' = \angle CO'E < \angle CO'D$.

We can verify these facts as before.

2.13 The converse of the above geometrical property is also true. In other words, in **congruent circles** (or in the same circle) those chords which cut off equal arcs or subtend equal angles at the centre are equal.

Let us suppose O is the centre of a circle and AB and CD are two chords of this circle such that $\text{arc } AB = \text{arc } CD$. It is to be verified that chord $AB =$ chord CD . You have seen (fig. 32) that under a special transformation the geometrical figure OAB along with the arc coincides with the geometrical figure OCD along with the arc. It can be found, despite different conditions that under the same transformation, chord $AB =$ chord CD .

Changing further the conditions if we take $\angle AOD = \angle COD$ then two geometrical figures will also coincide under the same transformation, or in other words it will be seen that chord $AB = \text{chord } CD$.

Applying the transformation, as before, in two congruent circles it will be found that if arc $AB = \text{arc } CD$, then chord $AB = \text{chord } CD$ and also chord $AB = \text{chord } CD$ if $\angle AOB = \angle CO'D$.

Exercise 3

1. Show with help of translation that when a line intersects two lines then the other two straight lines will be parallel to each other if a pair of corresponding angles are equal.

2. Show with the help of translation that both of two intersecting straight lines cannot be parallel to a third line.

3. What do you mean by congruent figures? Establish the congruence of two segments or two angles under reflection in two mutually intersecting straight lines.

4. Write down the first criterion for two triangles to be congruent. Establish this using translation, reflection or rotation.

5. Write the second criterion for two triangles to be congruent. Show it using translation, reflection or rotation.

6. When are two circles congruent? Using translation, rotation and reflection show that in congruent circles equal chords cut off equal arcs and subtend equal angles at the centre.

7. Using rotation and reflection show that in a circle the chords that cut off equal arcs or subtend equal angles are equal.

8. Using reflection show that the diameter of a circle divides it into two congruent semi-circles.

9. Draw an isosceles triangle ABC . Obtain the length of its base BC by measurement. Obtain the middle point D of BC . Will the $\triangle ABD$ and $\triangle ACD$, so formed, be congruent? (Hint : Reflect in the line AD).

Chapter 3

Various Constructions

3.1 To draw an angle equal to a given angle :

ABC is a given angle. An angle is to be drawn equal to this angle.

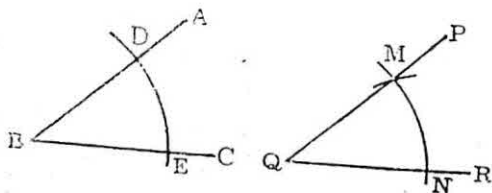


Fig. 38

Method of construction : Suppose that, QR is any line in the plane of the angle ABC. Draw an arc of a circle with any radius and the vertex B of the angle ABC as its centre. Suppose that the arc intersects BA and BC at D and E respectively. Now draw another arc of a circle with Q on the line QR as its centre and with the previous radius. Suppose that, it intersects QR at the point N. Now draw another arc with radius equal to ED and N as centre. Suppose that, this arc intersects the previous arc at M. Produce QM to P by a scale after joining QM. Then $\angle PQR$ is the angle equal to the given $\angle ABC$. Measure with the protractor to find that $\angle PQR = \angle ABC$.

Exercise 4

1. Draw an angle of 60° using protractor and draw an angle equal to it without using protractor.
 2. Bisect a right angle after drawing it using protractor. Now, draw another angle, without using protractor, equal to half the right angle and compute the value of the angle after measuring it with the protractor.
 3. Draw a triangle using a scale. Draw three angles at one point, in succession, equal to the three angles of the triangle. What is the sum of the three angles after measuring them with the protractor ?
 4. Draw an angle of 120° using protractor and an angle equal to it without using protractor.
-

3.2 Classification of triangles :

You need to know about classification of triangles before you start to draw different types of triangles.

A plane figure enclosed and formed by three straight lines is called a **triangle**.

A triangle has three sides and three angles. The three angular points are called vertices of the triangle. The classification of triangles is made on the basis of sides and angles.

On the basis of sides there are three types of triangles : **equilateral triangle**, **isosceles triangle** and **scalene triangle**.

Equilateral triangle : A triangle whose sides are of equal lengths is called an **equilateral triangle**.

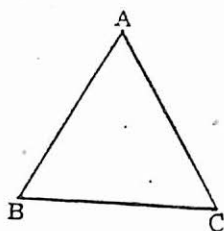


Fig. 39

In Fig. 39, in the equilateral triangle ABC, $AB = BC = CA$. Remember that the value of each angle of the triangle is the same and the value of each angle is 60° .

or $\angle A = \angle B = \angle C = 60^\circ$. Verify this fact using protractor.

Isosceles triangle : A triangle where two sides are of equal lengths is called an **isosceles triangle**.

In Fig. 40, in the isosceles triangle $AB = AC$. Remember that the base angles of an isosceles triangle are equal or $\angle B = \angle C$. Verify this fact by using protractor.

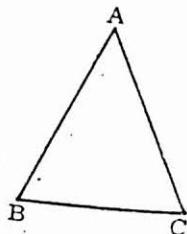


Fig. 40

Scalene triangle : A triangle where three sides are unequal in lengths is called a **scalene triangle**.

In Fig. 41, ABC is a scalene triangle. In a scalene triangle, the angles are also unequal. Verify this also by using protractor.

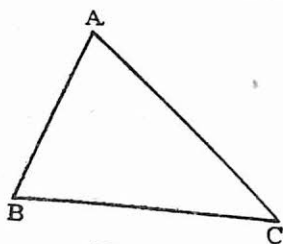


Fig. 41

Triangles are also divided on the basis of angles. Their names are **acute angled triangle**, **obtuse angled triangle** and **right angled triangle**.

Acute angled triangle : A triangle, where three angles are acute is called an **acute angled triangle**.

In Fig. 42, ABC is an acute angled triangle. Its each angle is an acute angle.

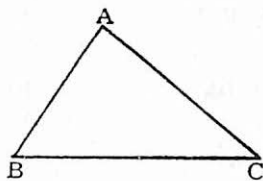


Fig. 42

[You know that an angle less than 90° is called an acute angle.]

Remember that equilateral triangle is an acute angled triangle; for the value of each angle is 60° which is less than 90° .

Obtuse angled triangle : A triangle one of whose angles is an obtuse angle is called an **obtuse angled triangle**.

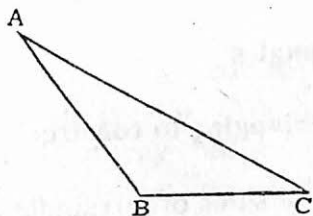


Fig. 43

In Fig. 43, in the obtuse angled triangle ABC, $\angle B$ is an obtuse angle.

[You know that an angle greater than 90° but less than 180° is called an obtuse angle.]

In an obtuse angled triangle, the two angles other than the obtuse angle are acute angles.

Right angled triangle : A triangle one of whose angles is a right angle is called a **right angled triangle**.

In Fig. 44, in right angled triangle ABC, $\angle B$ is a right angle.

[You know that if the value of an angle is 90° , it is called a right angle.]

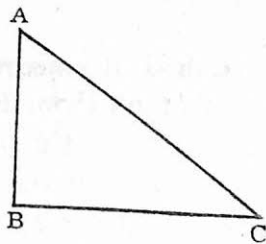


Fig. 44

The other two angles of a right angled triangle are acute angles.

Hypotenuse : The side opposite to the right angle of a right angled triangle is called **hypotenuse**.

In fig. 44, the side AC is the hypotenuse as it is opposite to the right angle $\angle B$ of the right angled triangle.

[Isosceles right angled triangle : If the adjacent sides of the right angle of a right angled triangle have equal lengths, then it is called an **isosceles right angled triangle**. In this case, each of the angles other than the right angle has the value 45° .] Triangles are classified into six types on the basis of sides and angles.

Construction of triangles

3.3 Given three sides of a triangle; to construct the triangle.

Given the lengths a, b, c of the sides of a triangle to construct the triangle.

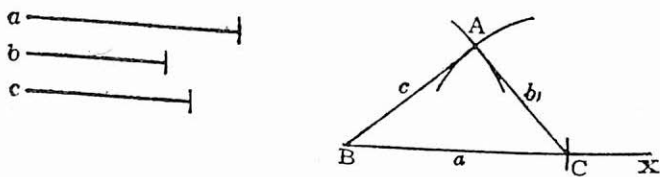


Fig. 45

Method of construction : Draw any straight line BX. Cut off from BX, the length BC equal to the length of given side a . Now draw an arc of a circle with B as centre and with radius equal to the given side c and draw another arc of a circle with C as centre and with radius equal to the given side b .

Suppose that the arcs intersect each other at A. Join AB and AC by scale. Then ABC is the required triangle.

Use the scale to verify that side $BC = a$, side $AC = b$ and $AB = c$.

N.B. : In this kind of construction of a triangle, values of the three line-segments may be given. In that case, one has to find out the length of BC and one has to take given lengths as radii for the construction of circular arcs.

[In general, the sides opposite to the angles A, B, C are denoted by a, b, c respectively.]

3.4 Given two sides and the included angle between them; to construct the triangle.

a, c are lengths of two sides and the included angle is $\angle x$. We have to construct the triangle in such a way that the lengths of two sides are equal to given a, c and the included angle equal to the given $\angle x$.

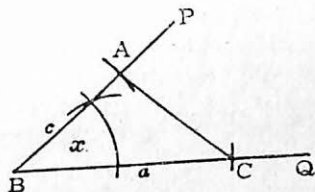
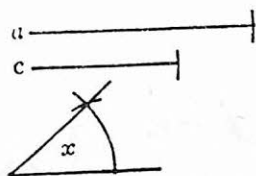


Fig. 46

Method of construction : BQ is any straight line. Cut off from BQ, the length BC equal to the side a . Now draw at B of BC, the angle $\angle CBP$ equal to the given angle $\angle x$. Cut off from BP, the portion BA equal to the given side c and join AC. Then

ABC is the required triangle where side $BC = a$, side $AB = c$ and $\angle ABC = \angle x$.

Use scale and protractor to verify measures of the sides and angles.

[In this case also, values of lengths of the sides and of the angles may be given. In that case, draw the triangle after drawing the line-segment with the scale and angles with protractor.]

3.5 Given the side of a triangle and the two angles adjacent to the side, construct the triangle.

Given the length of the side a and the two angles $\angle x$ and $\angle y$ adjacent to the side a . A triangle is to be constructed such that one of its sides is equal to the length of the side a and the measures of the angles adjacent to the side a are respectively equal to $\angle x$ and $\angle y$.

Method of construction : Take any straight line BQ. Cut off from BQ, the portion BC equal to the

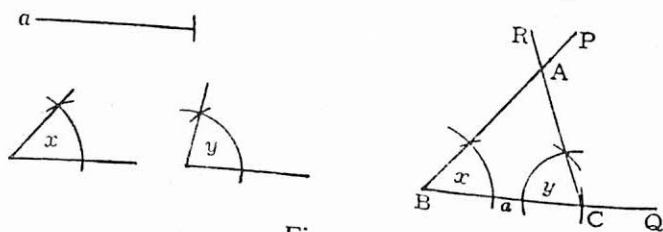


Fig. 47

side a . Draw at B of the side BC, the angle $\angle CBP$ equal to the given $\angle x$. Also, draw at C of the side BC, the angle $\angle BCR$ equal to the angle $\angle y$. Let BP and CR intersect each other at A. Then, ABC is the required triangle, whose side $BC = a$, $\angle ABC = \angle x$, $\angle ACB = \angle y$.

Use scale and protractor to measure sides and angles.

[Given the measures of sides and the adjacent two angles of a triangle; drawing the side and the two angles using scale and protractor, draw the triangle as before.]

Exercise 5

1. What is a triangle ? What are different types of a triangle on the basis of sides and angles ? Give definitions with figures for each type.

2. Can an equilateral triangle be called an isosceles triangle ? If so, give reasons. State the converse.

3. What is the hypotenuse of a right angled triangle ? When can this be called a right angled isosceles triangle ? Show it with figures.

4. Measure the angles of a triangle after drawing the triangle with sides 4.5 cm, 6 cm and 7.5 cm. Mention the type of the triangle.

5. The two sides of a triangle are of lengths 3.5 cm and 4.2 cm and the included angle between them is 60° . Draw the triangle.

6. The length of a side of a triangle is 8.2 cm, and the angles adjacent to the side are 60° and 45° . Draw the triangle.

7. Draw an equilateral triangle with 5.5 cm length of its sides. Obtain the measures of the angles of the triangle with protractor.

8. An isosceles triangle has two of its sides each of length 4.8 cm and the other side has the length 3 cm. Construct the triangle.

9. The base of a triangle has the length 6.5 cm and each of the two base angles of the triangle has the measure 45° . Draw the triangle and state the type of the triangle.

10. The lengths of sides of a right angled triangle containing the right angle are 3 cm and 4 cm. Draw the right angled triangle and obtain the measure of the length of the hypotenuse using scale.

11. The base of an isosceles triangle is 4.2 cm and the sum of the base angles is 100° ; draw the triangle.

3.6 Classification of quadrilaterals :

Quadrilateral : A plane figure enclosed by four line-segments is called a **quadrilateral**. A quadrilateral has four angular points or vertices. In fig. 48, the quadrilateral ABCD has four angular points A, B, C, D. A quadrilateral has four sides. Here, the four sides of the quadrilateral ABCD are AB, BC, CD, DA.

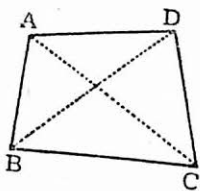


Fig. 48

The line-segment joining the opposite angular points is called **diagonal of the quadrilateral**. A quadrilateral has two diagonals. Here AC and BD are two diagonals.

The definitions together with figures of different quadrilaterals are given below:

Parallelogram : A quadrilateral whose opposite sides are parallel is called a **parallelogram**.

In Fig. 49, $AB \parallel DC$, $AD \parallel BC$; hence ABCD is a parallelogram. AC and BD are two diagonals of the parallelogram.

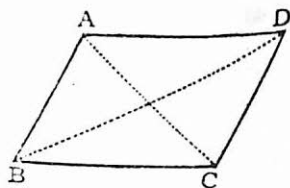


Fig. 49

N.B. : The opposite sides and angles of a parallelogram are equal. This means $AD = BC$, $AB = DC$; $\angle BAD = \angle BCD$, $\angle ABC = \angle ADC$. Also note that diagonals bisect each other.

Rectangle : A parallelogram with one of its angles equal to a right angle is called a **rectangle**.

In Fig. 50, $\angle B$ is the right angle of the parallelogram ABCD. Hence ABCD is a rectangle.

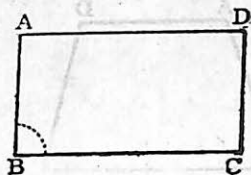
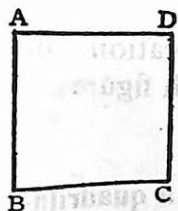


Fig. 50

N.B.: If one angle of a parallelogram is a right angle, then each angle of a rectangle is a right angle. It is clear from definition that any rectangle is a parallelogram, but every parallelogram is not a rectangle. The opposite sides of a rectangle are parallel and equal and the angles, a right angle. In the above rectangle, $AB \parallel DC$ and $AB = DC$, $AD \parallel BC$ and $AD = BC$, $\angle DAB = \angle ABC = \angle BCD = \angle CDA = 1$ right angle.

Square : A rectangle with its two adjacent sides equal is called a square.



In Fig. 51, in the rectangle ABCD, $AB = BC$; hence ABCD is a square. Clearly, $AB = BC = CD = DA$ and $\angle DAB = \angle ABC = \angle BCD = \angle CDA = 1$ right angle.

Fig. 51

N.B. : A square results from a rectangle under special circumstances; the square is also a parallelogram. Therefore, it can be said that all the properties of a parallelogram are true for a square.

Trapezium : A quadrilateral with a pair of parallel sides is called a trapezium.

In Fig. 52, in the quadrilateral ABCD, $AD \parallel BC$. Therefore ABCD is a trapezium. The two non-parallel sides of a trapezium are called **oblique sides**. In the trapezium ABCD, the two sides AB and DC are oblique sides.

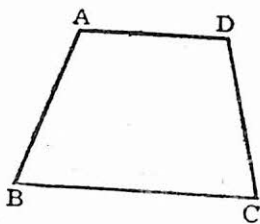


Fig. 52

Rhombus : If the sides of a quadrilateral are equal, but the angles are not equal to a right angle, it is called a **rhombus**.

In Fig. 53, in the quadrilateral ABCD, $AB = BC = CD = DA$; but no angle is a right angle. Hence ABCD is a rhombus.

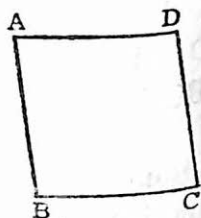


Fig. 53

N.B.: You will learn later that a rhombus is also a parallelogram. The angle included between two diagonals of a rhombus is 90° .

You have learnt about classification of quadrilaterals. Learn each definition with figures.

Construction of quadrilaterals

3.7 Given the lengths of four sides of a quadrilateral and the length of one of its diagonals, to construct the quadrilateral.

a, b, c, d are the lengths of four sides of the quadrilateral and e the length of the diagonal. A quadrilateral is to be so constructed that the lengths of its four sides are equal to the length of given four sides and length of one of its diagonals is equal to the length of the given diagonal.

Method of construction : Draw any straight line AX . Cut off from AX , the portion AC equal to the length of the diagonal. With A and C of the line-segment AC , as centres and with given

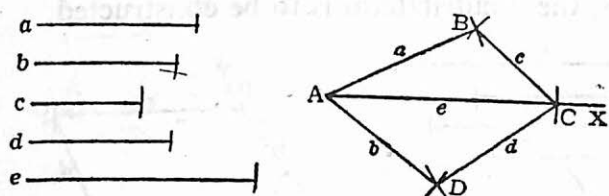


Fig. 54

lengths of sides a and c as radii, draw two arcs of circles. Let the two arcs intersect at the point B . Again, with A and C as centres draw, on the other side of AC , two other circular arcs, with given sides b and d as radii respectively. Let these two arcs intersect at the point D .

Join AB , AD , BC and CD . Then $ABCD$ is the required quadrilateral.

By construction (measuring with scale) for the quadrilateral $ABCD$, side $AB = a$, side $AD = b$, side $BC = c$, side $CD = d$ and diagonal $AC = e$.

N.B. : (i) Given the values of lengths of four sides and a diagonal, draw the quadrilateral, as before, after drawing the sides with the help of the scale.

(ii) It is necessary to point out which diagonal joins which vertices for the construction of the triangle. In the above construction, the diagonal AC has been taken as e . The figure would have been different if e was the diagonal BD .]

3.8 Given the lengths of four sides of a quadrilateral and one angle; to construct the quadrilateral.

Given a, b, c, d the lengths of four sides of a quadrilateral and the included angle $\angle x$ between a and b ; the quadrilateral is to be constructed.

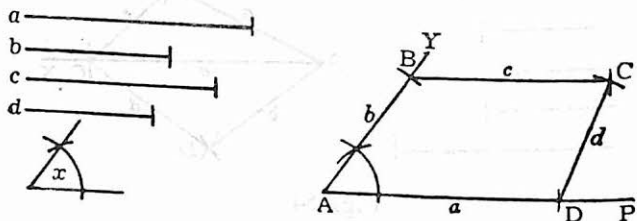


Fig. 55

Method of construction : Draw any straight line AP . Cut off from AP , the portion AD equal to the length of the given side a . Draw at A of the segment AD the $\angle DAY$ equal to the given $\angle x$. Cut off from AY , the portion AB equal to the given side b . Now draw with B and D as centres and with respective radius equal to given sides c and d , two arcs of circles. Suppose that the two circular arcs intersect at C . Join BC and CD . Then $ABCD$ is the required quadrilateral.

Measure with scale and protractor to find that in the quadrilateral $ABCD$, side $AD = a$, side $AB = b$, side $BC = c$, side $CD = d$ and $\angle BAD = \angle x$.

[N.B. : Given the measures of lengths of four sides and one angle of a quadrilateral, draw the quadrilateral as before after drawing by scale and protractor the sides and angle.]

3.9 Given the three sides and the two included angles between them; to construct the quadrilateral.

Given the three sides a, b, c of a quadrilateral, the included angle $\angle x$ between a and b , and the included angle $\angle y$ between c and a . The quadrilateral is to be constructed.

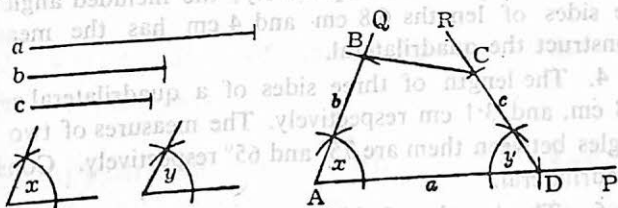


Fig. 56

Method of construction : Draw any straight line AP . Cut off from AP , the portion AD equal to the side a . Draw angles $\angle DAQ$ and $\angle ADR$ at the points A and D of the side AD , equal to $\angle x$ and $\angle y$ respectively. Now cut off from AQ , the portion AB equal to the length b and from DR , the portion DC equal to the length c . Join BC . Then $ABCD$ is the required quadrilateral.

Use scale and protractor to find that side $AD = a$, side $AB = b$, side $CD = c$; the included angle $\angle BAD$ between sides a and $b = \angle x$ and the included angle $\angle CAD$ between sides c and $d = \angle y$.

[N.B. : Given the measures of lengths of three sides and the measures of two included angles, draw the quadrilateral, as before, after drawing the three sides and the two angles using scale and protractor.]

Exercise 6

1. Give definition with figures :
Quadrilateral, parallelogram, rectangle, square, trapezium and rhombus.

2. The lengths of sides AB, BC, CD and DA of a quadrilateral ABCD are 5 cm, 5.2 cm, 5.4 cm, 5.6 cm respectively and that of its diagonal is 7.1 cm. Construct the quadrilateral.

3. The lengths of four sides of a quadrilateral are 6.8 cm, 4 cm, 4.2 cm, 4.5 cm respectively; the included angle between the sides of lengths 6.8 cm and 4 cm has the measure 55° . Construct the quadrilateral.

4. The length of three sides of a quadrilateral are 5.2 cm, 6.8 cm, and 3.1 cm respectively. The measures of two included angles between them are 75° and 65° respectively. Construct the quadrilateral.

5. The lengths of sides of the quadrilateral ABCD are the following: $AB=4.2$ cm, $BC=3.8$ cm, $CD=4$ cm, and $DA=2.9$ cm and the length of the diagonal $BD=6$ cm. Construct the quadrilateral. Bisect the sides AB, BC, CD, DA at four points E, F, G and H respectively. Join EF, FG, GH and HE. What is the relation among these sides? What can you then conclude about the quadrilateral EFGH?

Some necessary constructions

3.10 Construction of an angle 60° with scale and compasses :

Draw any straight line BC. With B as centre and with any radius, draw an arc of a circle. Let this arc meet BC at D. Now with D as centre and with the previous radius (or with radius equal to BD) draw another arc of a circle. Let this circular arc intersect the previous circular arc at E. Now produce BE to A after joining BE. Then the measure of $\angle ABC$ is 60° .

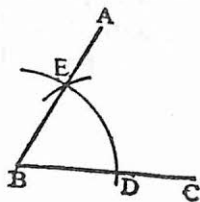


Fig. 57

Verify it with the help of a protractor.

[N.B.: If $\angle ABC$ is bisected, each angle will have the measure 30° and each angle will have the measure 15° if it is divided into four equal parts.]

3.11 Construction of angles 90° and 120° with scale and compasses :

BC is any straight line. With B of BC as centre and with any radius, draw a near semi-circular arc, such that it intersects BC at D. With D as centre and with the same radius draw an arc of a circle such that it intersects the previous circular arc at E. Again with E as centre and with the same radius, draw a circular arc such that it intersects the first circular arc at F. With E and F as centres and with the previous radius, draw two more arcs of circles. Let these two circular arcs intersect at G.

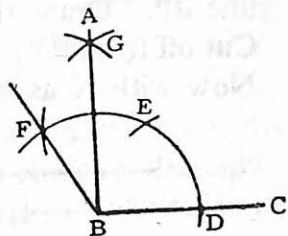


Fig. 58

Join BG and produce it to A. Then $\angle ABC = 90^\circ$.

Join BF and produce it. Then $\angle FBC = 120^\circ$.

Verify the measure of the angle in both the cases with protractor.

[N.B.: If $\angle ABC = 90^\circ$ is bisected, then measure of each angle is 45° . Try if you can draw angles with measures 75° and 135° from the above construction.]

Construction of triangles with some conditions

3.12 Given two sides and the opposite angle of one of these sides, construct the triangle.

Two sides c, b and $\angle x$, opposite to the side b , are given. A triangle is to be so constructed that lengths of its two sides are equal to given sides and angle opposite to the side b equal to given $\angle x$.

Method of construction : Draw any straight line BP. Draw the $\angle PBY$ at B equal to given $\angle x$. Cut off from BY, the portion BA equal to the side c . Now with A as centre and with given side b as

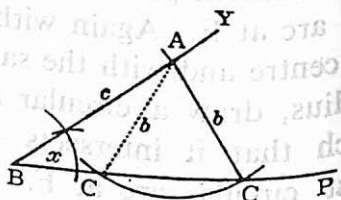
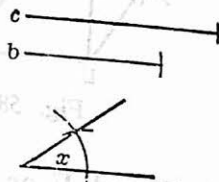


Fig. 59

radius, draw an arc of a circle. Let this circular arc intersect BP at C and C' respectively. Now join AC or AC' . Then $\triangle ABC$ or $\triangle ABC'$ is the required triangle, where side $AB = c$, side AC' or $AC = b$ and angle opposite to AC or $AC' = \angle ABC$ or $\angle ABC' = \angle x$.

[N.B.: If the length of the side b is small (less than the perpendicular length from A on BC), then the circular arc drawn with A as centre will not intersect BP.]

3.13 To draw a right angled triangle when the length of its hypotenuse and length of one of its other sides are given.

Let a be the hypotenuse and the length of the other side b be given. The right angled triangle is to be constructed.

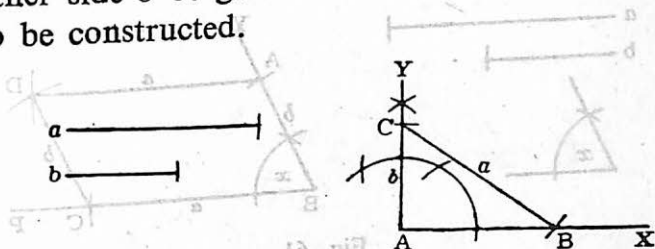


Fig. 60

Method of construction : Draw any straight line AX. Draw at A using compass, the right angle XAY. Cut off from AY, the portion AC equal to the given side b . Draw an arc of a circle with C as centre and the hypotenuse a as radius. Let the circular arc intersect AX at B. Join BC. Then ABC is the required right angled triangle, where $\angle BAC = 90^\circ$ and side BC opposite to the right angle = hypotenuse a ; the other side $AC = b$.

Construction of quadrilateral under some conditions

3.14 Given two adjacent sides and their included angle of a parallelogram to construct the parallelogram.

a, b are two adjacent sides and $\angle x$ is the included angle; the parallelogram is to be constructed.

Method of construction : Draw any straight line BP. Cut off from BP, the portion BC equal to the

given side a . Draw at B of BC, the $\angle CBY$ equal to the given $\angle x$. Cut off from BY, the portion BA equal to given side b . Now draw with A and C as centres and respective radius equal to sides a and b ,

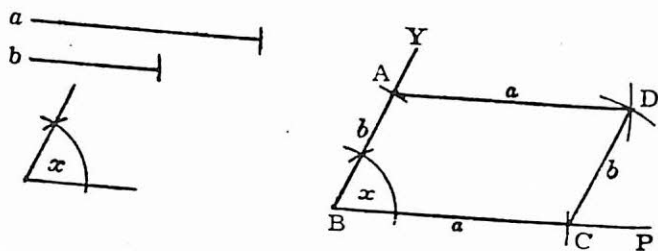


Fig. 61

two circular arcs. Let these two circular arcs intersect at D. Join AD and CD. Then ABCD is the required parallelogram, for $BC = AD = a$, $AB = CD = b$ and the included $\angle ABC$ between AB and BC = $\angle x$.

[N.B.: A right angle ($=90^\circ$) drawn at B will make the parallelogram a rectangle.]

Exercise 7

1. The lengths of adjacent sides of a parallelogram are 7.5 cm and 3.5 cm respectively and the measure of an angle is 60° . Construct the parallelogram.
2. The lengths of adjacent sides of a rectangle are 6 cm and 4 cm respectively. Construct the rectangle.
3. The lengths of adjacent sides AB and AD of a parallelogram are 4 cm and 6.5 cm respectively and the length of the diagonal BD is 7 cm. Construct the parallelogram.

3.15 Construct a square with a given side.

a is a given side. A square is to be constructed with a as its side.

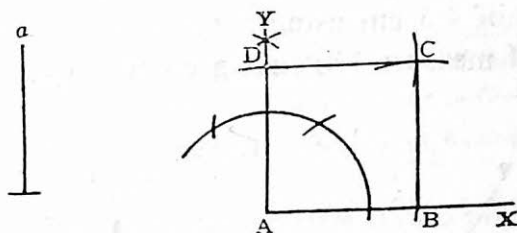


Fig. 62

Method of construction : Draw any straight line AX . Cut off the portion AB from AX equal to the given side a . Draw at A of AB , the right angle $\angle BAY$. Cut off the portion AD from AY equal to the side a . Again, with B and D as centres, draw two more circular arcs with radius equal to the side a . Let the two circular arcs intersect at C . Join BC and CD . Then $ABCD$ is the square where each side has the length equal to the given length a .

Exercise 8

1. Draw a square with 4.6 cm as its side.
2. The length of a diagonal of a square is 5.8 cm. Construct the square.
[Hint : The diagonal of a square bisects its angles.]

3.16. The length of a side of a rhombus is 4.5 cm and the measure of an angle is 120° ; draw the rhombus.

Method of construction : First, draw a straight line a of 4.5 cm using a scale and draw the angle $\angle x$ of measure 120° using compasses.

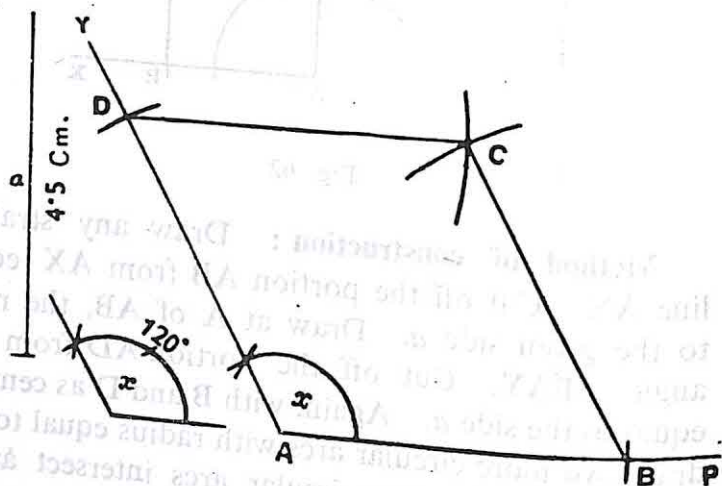


Fig. 63

Cut off the portion AB from any straight line AP , equal to the side a (4.5 cm). At A , draw $\angle BAY$ equal to given $\angle x = 120^\circ$. Again cut off length AD equal to a (4.5 cm) from AY . Now with B and D as centres draw the circular arcs with each radius equal to 4.5 cm. Let the two circular arcs meet at the point C . Join BC and CD . Then $ABCD$ is the required rhombus, whose each side is equal to 4.5 cm. and $\angle BAD = 120^\circ$.

Exercise 9

1. The length of a side of a rhombus is 5.2 cm and measure of an angle is 75° . Construct the rhombus.

2. Construct a rhombus with side 6.5 cm and with an angle whose measure is 60° ; obtain the measure of the included angle between two diagonals after drawing them first.

3. The lengths of diagonals of a rhombus are 4 cm and 5 cm. Construct the rhombus.

[Hint : The diagonals bisect each other at right angles.]

Answers

Exercise 4

2. 45° 3. 180°

Exercise 5

4. Right angled triangle 10.5 cm.

Exercise 6

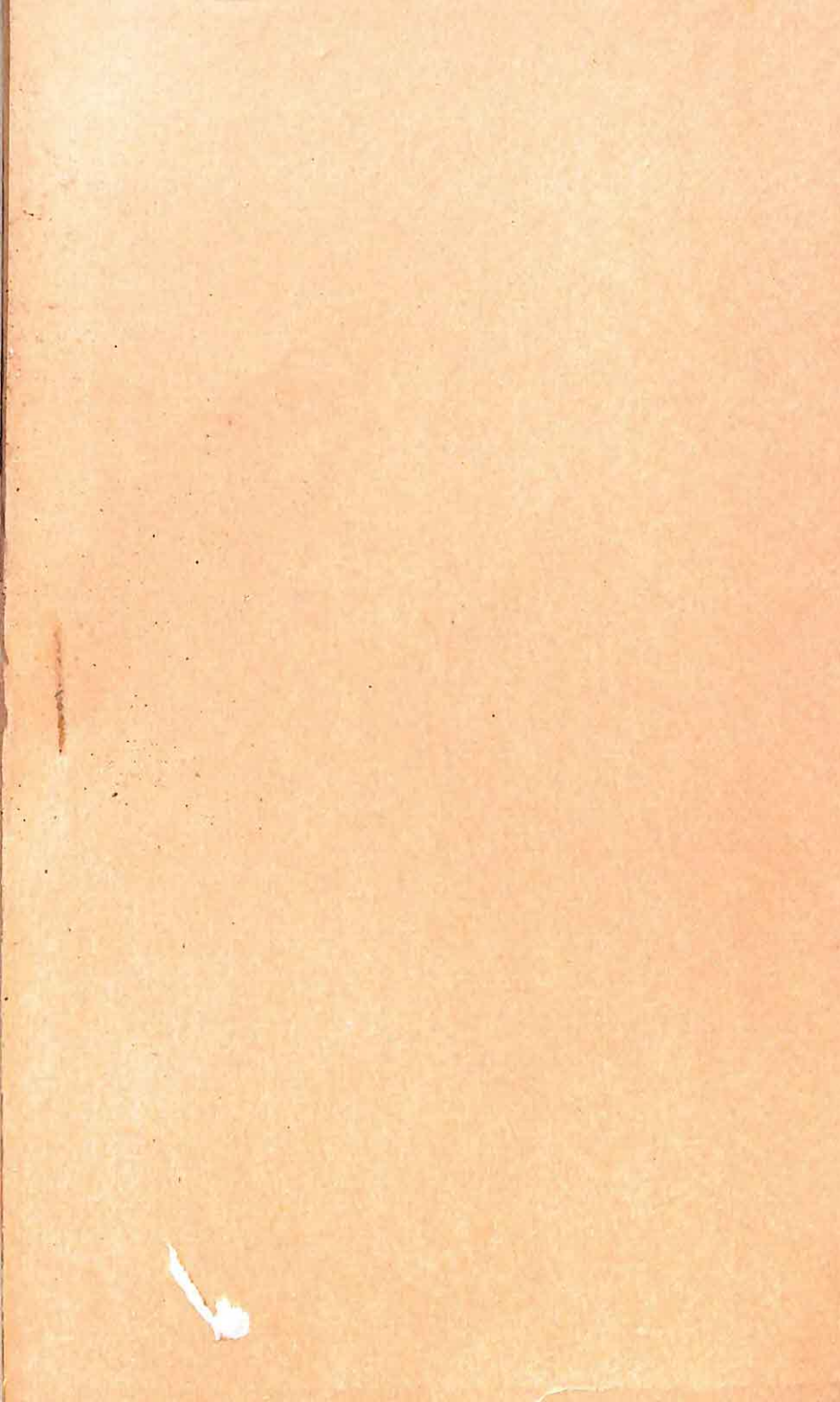
5. $EF=GH$ and $FG=HE$; the quadrilateral EFGH is a parallelogram.

Exercise 9

2. 90° .
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